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A method for the construction of hierarchical generalized space shift keying (GSSK) modulation for unequal error protection

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A B S T R A C T

In this paper, we propose a systematic method to achieve two-level unequal error protection (UEP) with generalized space shift keying (GSSK) modulation for multiple-input multiple-output (MIMO) systems. GSSK is a modulation scheme that encodes the source information in the antenna indices. To enable the nonuniform arrangement of the spatial constellation of GSSK modulation, new techniques are needed as new features of GSSK modulation are observed. The proposed method is based on first partitioning the GSSK constellation into subsets and then choosing constellation points inside a subset. The high-priority (HP) bits select the subset and the low-priority (LP) bits select constellation points inside a subset. The proposed subset partitioning procedure developed based on a theoretical study on set partitioning in the multidimensional space guarantees optimal or near-optimal HP protection, enables simple and systematic designs for LP protection, and increases the LP capacity. The proposed systematic design approach applies to systems with any numbers of transmit antennas and can create various hierarchical GSSK schemes with different HP/LP protection capabilities.

1. Introduction

Common unequal error protection (UEP) techniques for communications include the use of different rates of error-correction codes and nonuniform signal constellation arrangements to provide different levels of data protection. The fundamental idea underlying nonuniform constellations is to assign larger Euclidean distances to signal points addressed by the important bits than to those addressed by the less important bits. This strategy was extensively studied in the context of uncoded modulations such as pulse/quadrature amplitude modulation (PAM/QAM) [1] and phase shift keying (PSK) [2]. UEP was also studied in the context of combined coding and modulation [3–8], where the construction of coded modulations for UEP is achieved by signal set partitioning and code selection. Modulations with UEP capabilities have found applications in multimedia communications [9–11], broadcasting [12,13], and relay communications [14].

Recently, spatial modulation (SM) [15] has emerged as a promising transmission technique that specifically exploits the deployment of multiple antennas in multiple-input multiple-output (MIMO) wireless communication systems. Unlike conventional phase and amplitude modulations, SM uses both the antenna indices and the conventional data symbols to carry information. A low-complexity implementation of SM, i.e., space shift keying (SSK) [16], encodes the source information entirely in the varying indices of a single activated transmit antenna. Generalized SSK (GSSK) modulation [17] generalizes SSK by allowing a fixed number of two or more activated transmit antennas
to improve the transmission rates of SSK. Hamming code-aided SSK (HSSK) [18] suggests the use of varying numbers of activated transmit antennas according to the Hamming code patterns. Space–time SSK schemes [19–21] extend SSK to both space and time dimensions by combining SSK with space–time block coding (STBC). For a more extensive coverage of this subject, see the tutorial in [22].

Enabling UEP with SSK-type modulations is of tremendous interest for future wireless MIMO systems employing SSK-type modulations for multimedia delivery. Encoding the source information fully in the antenna indices, as in SSK-type modulations, essentially creates an entirely different constellation as compared to PSK/PAM/QAM. This suggests that a new approach is needed to realize the nonuniform constellation placement. In [23], UEP design concepts inspired by code construction in accordance with the HSSK design principle were presented. In this contribution, we conduct a further investigation of UEP with SSK-type modulation from a theoretical perspective. In particular, we consider an uncoded MIMO system employing the GSSK modulation. The proposed hierarchical GSSK (H-GSSK) scheme incorporates two elements corresponding to two levels of protection: (1) a systematic method developed based on a theoretical study on set partitioning in the multidimensional space to achieve optimal or near-optimal first-level (high-priority, or HP) protection, and (2) graph-based methods to select spatial constellation points to accomplish second-level (low-priority, or LP) protection. Simulation demonstrates the UEP performance of the proposed H-GSSK while achieving the same aggregate rate as the nonhierarchical GSSK, with comparable hardware requirements.

This paper is organized as follows. Section 2 presents the system model and preliminaries on hierarchical modulation. The proposed methods are described in Section 3. Design examples are presented in Section 4 and performance results in Section 5. Conclusion is given in Section 6.

2. Preliminaries

2.1. System model

We consider an uncoded MIMO system with $N_T$ transmit antennas and $N_R$ receive antennas (an $N_T \times N_R$ system). The system employs the GSSK modulation. The complex baseband transmission model is

$$y = H\sqrt{E_s}\tilde{x} + v$$

(1)

where $y \in \mathbb{C}^{N_R \times 1}$ is the received signal, $\tilde{x}$ is the $N_T \times 1$ GSSK-modulated symbol, $H \in \mathbb{C}^{N_R \times N_T}$ is the flat-fading channel, $v \in \mathbb{C}^{N_R \times 1}$ is the additive white Gaussian noise (AWGN), and $E_s$ is the transmit power at each antenna. Channel matrix $H$ has independent and identically distributed (i.i.d.) complex Gaussian entries with zero mean and covariance matrix $\sigma_H^2 I_{N_T}$, where $I_{N_T}$ is the identity matrix with dimension specified in the subscript. Perfect channel information is assumed at the receiver. Noise $v$ has i.i.d. complex elements with zero mean and covariance matrix $(N_0/2)I_{N_R}$. Transmitted GSSK symbol (constellation point) $\tilde{x}$ is a binary vector comprised of fixed numbers of 1’s (corresponding to activated antennas) and 0’s (corresponding to idle antennas), i.e.,

$$\tilde{x} = \begin{bmatrix} 1, 0, \ldots, 0, 1, \ldots, 0 \end{bmatrix}^T.$$

(2)

The transmitted symbol is selected equiprobably from the GSSK modulation alphabet (constellation set) $A$. The value of $n_t$ is chosen to be the minimum number of 1’s required to encode the given amount of information, i.e., $\left( \frac{N_T}{n_t-1} \right) < |A| = 2^m \leq \left( \frac{N_T}{n_t} \right)$, where $n_t \leq N_T/2$ for $m$ bits/s/Hz transmission, where $U$ is the set of all available antenna indices (constellation universe) and $|\cdot|$ is the cardinality of a set. The required number of transmitter radio frequency (RF) chains for GSSK modulation is specified by $n_t$. SSK is a special case of GSSK with $n_t = 1$.

Given the signal model in (1), the average symbol-error-rate (SER) performance of the system based on the optimal maximum likelihood (ML) detection can be quantified by first deriving the pairwise error probability (PEP) and then using the union bound technique [17,24]. The SER $P_s$ is shown to be bounded by

$$P_s \leq \frac{1}{|A|} \sum_{x_j \in A} \sum_{\tilde{x} \in A} \frac{1}{2} \left( \frac{E_s}{2N_0N_T} \right)^{-N_R} = d(\tilde{x}, x_j)^{-N_R} \quad (3)$$

where $d(\tilde{x}, x_j)$ is the Hamming distance between distinct symbols $\tilde{x}$ and $x_j$. As can be seen, at a given $E_s/N_0$ the dominant terms on the right-hand-side of (3) correspond to symbol pairs with small Hamming distances. Thus, at a given $E_s/N_0$ the performance of a GSSK-modulated system is dominated by the minimum Hamming distance among all distinct symbol pairs. It can easily be shown that the Hamming distances between arbitrary two distinct GSSK symbols are always positive even numbers.

2.2. Hierarchical modulation

UEP for conventional phase and amplitude modulations adopts a nonuniform constellation placement technique. Fig. 1(a) shows an example of hierarchical 4-PAM (4-HPAM) where the signal constellation is divided into two subsets to realize the two-level protection or hierarchy. $d_H^{(HP)}$ represents the Euclidean distance between the fictitious black points in the first level of hierarchy, and $d_L^{(LP)}$ represents the Euclidean distance between the actual transmitted white points in the second level of hierarchy. Adjusting the ratio $d_H^{(HP)} / d_L^{(LP)}$ customizes the protection of the HP bit (the left-most bit) with respect to the LP bit. The same design strategy is easily extendable to the two-dimensional QAM [1].

The performance results for GSSK in (3) suggest that we may consider a similar strategy of nonuniform constellation placement yet in terms of the Hamming distance (hereafter, the distance) to implement H-GSSK. However, new considerations may be needed due to the two new features of GSSK modulation [23]: (1) the spatial constellation of GSSK is $N_T$-dimensional as opposed to 1-D/2-D in PAM/QAM, and (2) the adjustment of the distance ratio is
subject to the number of transmit antennas and the set of available antenna indices, and thus has limited possibilities.

One objective of this work is to introduce a systematic method to implement H-GSSK that reflects these new features. Fig. 1(b) shows an example of H-GSSK where the spatial constellation is divided into subsets similar to Fig. 1(a). $d^{\text{HP}}$ represents the distance between the center of subsets (black points) in the first level of hierarchy, and $d^\text{(LP)}$ represents the distance between the elements inside a subset (white points) in the second level of hierarchy. A total of $|A| = 16$ GSSK symbols (both black and white points) are mapped by 1 HP bit (the left-most bit) and 3 LP bits. A real system represented by this example is one with $N_T = 6$ and $n_t = 3$ supporting 1 HP and 3 LP bits (see Section 4.1).

3. The proposed method for constructing H-GSSK

The objective of this work is to propose a systematic method to realize the concept of nonuniform constellation placement as shown in Fig. 1(b) for given values of $N_T$ (antenna constraint), $n_t$ (RF chains constraint), and $c^\text{(HP)}$ and $c^\text{(LP)}$ (capacity or rate constraint), where $c^\text{(·)}$ stands for the capacity or rate (i.e., bits per GSSK symbol transmission) of the corresponding bit priority.

3.1. First-level (high-priority) protection

In the first level of hierarchy, the center of subsets (“black points”) are determined. The design method is developed based on a theoretical result on set partitioning.

3.1.1. Optimal set partitioning

Let $\mathcal{U}^T$ be the $N_T$-dimensional binary symbol universe, i.e., $\mathcal{U}^T = \left\{ [x_1, x_2, \ldots, x_N] \mid x_i \in \{0, 1\}, \ i = 1, \ldots, N_T \right\}$, and $\mathcal{U}_n$ ($n = 0, 1, \ldots, N_T$) be a subset of $\mathcal{U}^T$ whose symbols are elements containing exactly $n$ 1’s, i.e., $\mathcal{U}_n = \left\{ [x_1, x_2, \ldots, x_N] \mid \sum_{i=1}^{N_T} x_i = n \right\}$. Clearly, $\mathcal{U}^T = \mathcal{U}_0 \cup \mathcal{U}_1 \cup \cdots \cup \mathcal{U}_{N_T}$. For GSSK modulation with parameter $n_t$, its constellation universe is given by $\mathcal{U} = \mathcal{U}_{n_t}$. Since $\mathcal{U} \subseteq \mathcal{U}^T$, it is useful to first examine the selection of the center of subsets (“black points”) in $\mathcal{U}^T$. A partitioning of $\mathcal{U}^T$ into subsets is called optimal set partitioning if the minimum distance between the center of different subsets is maximized.

**Theorem 1** (Optimal Inter-Subset-Center Distance). The minimum pairwise distance between arbitrary $c$ symbols in $\mathcal{U}^T$ that form the center of $c$ subsets, where $c$ is an integer greater than or equal to 2, is upper bounded by

$$d_{\text{opt}}(N_T, c) = \frac{N_T}{2} \left[ \left\lfloor \frac{c}{2} \right\rfloor + \left\lceil \frac{c}{2} \right\rceil \right]$$

where $\lfloor x \rfloor$ and $\lceil x \rceil$ are the floor and ceiling functions, respectively.

**Proof.** Let the minimum and average pairwise distances between any selected $c$ symbols in $\mathcal{U}^T$ be $d_{\text{min}}(N_T, c)$ and $d_{\text{avg}}(N_T, c)$, respectively, and denote the $i$th ($i = 1, 2, \ldots, c$) symbol by $a_i = [a_{i,1}, a_{i,2}, \ldots, a_{i,N_T}]^T$. Then, we have

$$d_{\text{min}}(N_T, c) \leq d_{\text{avg}}(N_T, c)$$

where $\oplus$ represents modulo-2 addition. Let $a_i^T$ be the $i$th row of a $c \times N_T$ binary matrix $B$. In deriving the second inequality of (5), note that $\sum_{i,j,k} a_{i,k} \oplus a_{j,k}$ is the sum of distance between all pairs of elements in the $k$th column of $B$. This sum value is given by $x(c - x)$ when the $k$th column of $B$ is comprised of $x$ 1’s and $c - x$ 0’s, since only pairs of 1 and 0 will contribute a nonzero value to the sum. The maximum of $x(c - x)$ is $\left\lfloor \frac{c}{2} \right\rfloor \cdot \left\lceil \frac{c}{2} \right\rceil$, achieved when $x = \left\lfloor \frac{c}{2} \right\rfloor$. Thus, we obtain the second inequality and complete the proof. □

As can be seen in (5), the upper bound is achieved when both inequalities become equalities. This happens when the distances between any two rows of $B$ are equal (first inequality becomes an equality) and when each column of
B contains exactly $\left\lfloor \frac{c}{i} \right\rfloor$ 1’s and $\left\lceil \frac{c}{i} \right\rceil$ 0’s (second inequality becomes an equality). An example of B having these two properties is

$$B = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \tag{6}$$

The rows of this matrix indicate the $c = 2$ symbols\(^1\) of length $N_T = 3$ with distance achieving $d_{\text{opt}}(3, 2) = 3$. These two symbols, however, contain different numbers of 1’s and therefore cannot be used as GSSK symbols. For B to be used to select GSSK symbols, rows of B must also contain the same number of 1’s. In other words, the c symbols selected from $U^i$ that achieve the theoretical optimum given in Theorem 1 must simultaneously lie in the subset $U = U_{n_t}$ for some $n_t$. For the previous example in (6), the optimal distance is achieved with one symbol in $U_1$ and the other in $U_2$. Two symbols that simultaneously lie in $U_1$ (or $U_2$) will result in a suboptimal distance equal to 2. Summarizing our discussions, we have the following axiom.

**Axiom 1 (Optimal Generator Matrix).** A $c \times N_T$ binary matrix G is an optimal generator matrix for generating $c$ GSSK symbols from its $c$ rows that collectively achieve optimal set partitioning of $U$, if the matrix satisfies the following three conditions simultaneously:

1. **(Validity)** Each row of G contains the same number of 1’s.
2. **(Optimality)** Each column of G contains $\left\lfloor \frac{c}{i} \right\rfloor$ 1’s and $\left\lceil \frac{c}{i} \right\rceil$ 0’s.
3. **(Symmetry)** The distances between any two rows of G are equal.

Note that Conditions 2 and 3 follow from the proof of Theorem 1, and Condition 1 ensures that the selected c symbols in $U^i$ are also in $U$.

### 3.1.2. Design method for specific $N_T$, $n_t = N_T/2$, and $C^{(HP)}$

It is possible to systematically create a matrix that satisfies the three conditions in Axiom 1. Specifically, if we take a $c \times 1$ vector comprised of $\left\lfloor \frac{c}{i} \right\rfloor$ 1’s and $\left\lceil \frac{c}{i} \right\rceil$ 0’s and let the possible element permutations of this vector be the columns of a matrix, the resulted $c \times \left( \begin{pmatrix} c \\ \left\lfloor \frac{c}{i} \right\rfloor \end{pmatrix} \right)$ matrix satisfies the three conditions in Axiom 1. For example,

$$G_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{7}$$

is a $2 \times 2$ optimal generator matrix and can generate $c = 2$ symbols from its rows for a system with $N_T = 2$ whose minimum distance achieves $d_{\text{opt}}(2, 2) = 2$, and

$$G_4 = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \tag{8}$$

is a $4 \times 6$ optimal generator matrix and can generate $c = 4$ symbols from its rows for a system with $N_T = 6$ whose minimum distance achieves $d_{\text{opt}}(6, 4) = 4$. A $c \times N_T$ optimal generator matrix has the property that the number of 1’s in each row is equal to $N_T/2$, and thus the symbols generated have $n_t = N_T/2$. A horizontal concatenation of an optimal generator matrix any times will result in another optimal generator matrix of a bigger dimension. Note that the interested case is $c$ being a power of two, since $c = 2^{C^{(HP)}}$ in our application. In addition, $C^{(HP)} = 1$ or 2 is of particular interest as it is the typical case in the conventional hierarchical PAM/QAM. The systematic design for specific $N_T$, $n_t = N_T/2$, and $C^{(HP)}$ is summarized as follows:

- $C^{(HP)} = 1 (c = 2)$: For an antenna system with $N_T = 2k$ ($k \in \mathbb{Z}^+$, where $\mathbb{Z}^+$ is the set of positive integers), generate $c = 2$ symbols from the rows of an optimal generator matrix G which is a horizontal concatenation of $G_2 k$ times.
- $C^{(HP)} = 2 (c = 4)$: For an antenna system with $N_T = 6k$ ($k \in \mathbb{Z}^+$), generate $c = 4$ symbols from the rows of an optimal generator matrix G which is a horizontal concatenation of $G_4 k$ times.

### 3.1.3. Design method for general $N_T$, $n_t$, and $C^{(HP)}$

When $N_T$ is too small or $C^{(HP)}$ is too big such that $N_T$ is not a positive multiple of $\left( \begin{pmatrix} c \\ \left\lfloor \frac{c}{i} \right\rfloor \end{pmatrix} \right)$, or when there is a constraint $n_t < N_T/2$, the previously described design method is not directly applicable. In this case, the optimum stated in Theorem 1 cannot be achieved by c symbols in $U = U_{n_t}$. The suboptimality is a result of either inequality in (5) being a strict inequality, or both. To facilitate the design for LP protection, we allow the second inequality to become a strict inequality but not the first (i.e., the optimality condition in Axiom 1 is relaxed but the symmetry condition is retained). More specifically, we want to construct a $c \times N_T$ binary suboptimal generator matrix G which satisfies the validity and symmetry conditions but not the optimality condition. For a given value of $c$, a set of matrices $G_{c0}, G_{c1}, \ldots, G_{c\left\lceil \frac{c}{i} \right\rceil}$ is constructed, where $G_{ci}$ is a $c \times \left( \begin{pmatrix} i \\ \left\lfloor \frac{c}{i} \right\rfloor \end{pmatrix} \right)$ matrix whose columns are comprised of all possible element permutations of a $c \times 1$ vector containing $i$ 1’s and $c - i$ 0’s. These matrices share the common property of validity and symmetry conditions being satisfied. For example, for $c = 4$, the set of matrices is given by

$$G_{40} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad G_{41} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{9}$$

$$G_{42} = G_4 \quad \text{in (8)}.$$

The $c$ symbols are then generated from the rows of a suboptimal generator matrix G which is a horizontal concatenation of possibly several matrices chosen from $G_{ci}$ ($i = 0, 1, \ldots, \left\lfloor \frac{c}{i} \right\rceil$). Suppose that matrix $G_{ci}$ is concatenated $g_i$ times in forming G. Then, our objective is to find $g_i$'s such

\(^1\) For notational convenience, we use $[1 \ 0 \ 1]$ and $[1, \ 0, \ 1]^T$ interchangeably to denote a symbol throughout this paper.
that the distance between the $c$ symbols is maximized. Formally, this is to solve

$$\max_{g_0, g_1, \ldots, g_{\left\lfloor \frac{c}{2} \right\rfloor}} \left\{ \sum_{i=1}^{\left\lfloor \frac{c}{2} \right\rfloor} 2 \left( c - 2 \right) g_i \right\}$$

s.t. (1) \( \sum_{i=0}^{\left\lfloor \frac{c}{2} \right\rfloor} (c) g_i = N_T \)

(2) \( \sum_{i=1}^{\left\lfloor \frac{c}{2} \right\rfloor} \left( c - 1 \right) g_i \leq n_t \)

(3) \( g_i \in \{Z^+, 0\}, \quad i = 0, 1, \ldots, \left\lfloor \frac{c}{2} \right\rfloor \).

In (10), 2 \( (c-2) \) in the objective function is the distance between any two rows of \( G_i \). The first constraint ensures that the generator matrix will have the desired dimension \( c \times N_T \). The second constraint reflects the RF chains constraint by limiting the number of 1’s in each row of the generator matrix to \( n_t \), where \( \left\lfloor \frac{c}{2} \right\rfloor \) is the number of 1’s in each row of \( G_i \). The second constraint can be disabled by letting \( n_t = N_T/2 \). Note that when \( c > N_T \) or equivalently \( C^{HP} > \log_2(N_T) \), the solution to (10) is \( g_0 = N_T \) and \( g_1 = \cdots = g_{\left\lfloor \frac{c}{2} \right\rfloor} = 0 \), i.e., the generator matrix being a zero matrix. This means that the specified value of \( C^{HP} \) is infeasible for the antenna system with \( N_T \).

The systematic design for general \( N_T \), \( n_t \), and \( C^{HP} \) is summarized as follows:

- For an antenna system with \( N_T \) and maximum required \( n_t \), generate \( c = 2^{C^{HP}} \) symbols from the rows of a suboptimal generator matrix \( G \) which is a horizontal concatenation of \( G_i \) \( c \) times \( (i = 0, 1, \ldots, \left\lfloor \frac{c}{2} \right\rfloor) \), where \( g_i \)’s are given by the solution to (10).

We plot the distance between the \( c = 2^{C^{HP}} \) symbols (the center of \( c \) subsets) selected based on the proposed method versus \( N_T \) in Fig. 2, for \( C^{HP} = 1 \) and \( C^{HP} = 2 \). As can be seen, for specific values of \( N_T \) (i.e., multiples of 2 and 6 for \( C^{HP} = 1 \) and \( C^{HP} = 2 \), respectively), the theoretical optimum is achieved by the proposed H-GSSK scheme; for other values of \( N_T \), the theoretical optimum cannot be achieved but is approached by the proposed scheme. It is seen that for a given \( c, d^{HP} \) is a positive even number that grows nondecreasingly with \( N_T \) for H-GSSK, whereas the theoretical optimum grows linearly with \( N_T \) (Theorem 1).

### 3.2. Second-level (low-priority) protection

Following the partitioning of \( \mathcal{U} \) into subsets in the first level of hierarchy, the elements inside the subsets (“white points”) are selected in the second level of hierarchy to accomplish the UEP design. Two selection methods are proposed.

#### 3.2.1. The sphere method

In the first method, a sphere of radius \( r \) (measured in terms of the Hamming distance) is drawn around each subset center (“black point”). The sphere radius \( r \) is chosen as the maximum positive even number such that different spheres do not overlap in \( \mathcal{U} \). The systematic design steps are summarized as follows:

1. Construct a sphere of radius \( r \) that centers at each subset center, where \( r \) is the maximum even number such that \( 2r < d^{HP} \).
2. Select \( 2^{C^{LP}} \) symbols inside each sphere of radius \( r \) (including those on the boundary). When the selection is not unique, it is done arbitrarily. When there are fewer than \( 2^{C^{LP}} \) symbols inside each sphere, this means that the specified value of \( C^{LP} \) is infeasible.
3. Label each selected symbol by a distinct string of \( C^{HP} + C^{LP} \) bits, where the HP bits map to different subsets and the LP bits map to different symbols in each subset.

Note that the total number of symbols inside a sphere of radius \( r \) is easily determined by the sum number of symbols on concentric spheres of radii 2, 4, \ldots, \( r \) plus the common center of these spheres, as depicted in Fig. 3. Symbols on the sphere of radius \( 2k \) can be identified by switching the positions of \( k \)’s and \( k \)’0’s of the “black point” which is the center of the sphere. The symmetry property maintained in the first level of hierarchy ensures the equal number of symbols inside each of the \( 2^{C^{HP}} \) spheres and thus facilitates the selection in the second level. In addition, maximizing the inter-subset-center distance in the first level enlarges the capacity of the LP bits.

#### 3.2.2. The Voronoi graph method

In the second method, a Voronoi graph of \( \mathcal{U} \) is created. The Voronoi cell \( V_i \) \( (i = 1, \ldots, 2^{C^{HP}}) \) associated with the
4. Design examples

4.1. Example A

We consider the system with $N_T = 6$ and the target sum rate of 4 bits/s/Hz (1 HP and 3 LP bits for H-GSSK). Following the proposed design steps, the two symbols ("black points") $[0, 1, 0, 1, 0, 1]^T$ and $[1, 0, 1, 0, 1, 0]^T$ are first selected which achieve the optimal $d^{(HP)} = d_{opt}(6, 2) = 6$. Suppose that we adopt the sphere method for the second-level design. With $d^{(HP)} = 6$, the maximum sphere radius without incurring a sphere overlap is $r = 2$. Among the nine symbols on the sphere of radius 2, seven symbols ("white points") are arbitrarily selected. These seven symbols along with the center of the sphere are labeled by $0000–0111$ (or $1000–1111$), where the HP bit is marked in boldface, as shown in Table 1. Note that Fig. 1(b) is an illustration of the sixteen H-GSSK symbols in Table 1 in the hyperspace with $d^{(HP)} = 6$ and $d^{(LP)} = 2$. The sixteen symbols of the nonhierarchical GSSK supporting the same sum rate are also shown in Table 1 for comparison, which are constructed based on plain lexicographic selection (i.e., varying the positions of 1’s from right to left). Both H-GSSK and nonhierarchical GSSK require the same number of transmitter RF chains ($n_t = 3$).

4.2. Example B

We consider the system with $N_T = 8$ and the target sum rate of 5 bits/s/Hz (1 HP and 4 LP bits for H-GSSK). First, the two symbols ("black points") $[0, 1, 0, 1, 0, 1, 0, 1]^T$ and $[1, 0, 1, 0, 1, 0, 1, 0]^T$ are selected which achieve the optimal $d^{(HP)} = 8$. As a result, the maximum sphere radius is $r = 2$. Then, the center of the sphere and the fifteen arbitrarily selected symbols on the sphere of radius 2 are labeled by $00000–01111$ (or $10000–11111$). H-GSSK

**Table 1**

<table>
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<th>H-GSSK symbols</th>
<th>Source bits</th>
<th>GSSK symbols</th>
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<td>[1, 0, 1, 0, 0, 0]^T</td>
<td>1001</td>
<td>[1, 1, 0, 0, 0, 1]^T</td>
</tr>
<tr>
<td>1010</td>
<td>[0, 0, 1, 1, 1, 0]^T</td>
<td>1010</td>
<td>[0, 0, 1, 1, 1, 0]^T</td>
</tr>
<tr>
<td>1011</td>
<td>[0, 0, 1, 1, 1, 1]^T</td>
<td>1011</td>
<td>[1, 0, 1, 0, 1, 0]^T</td>
</tr>
<tr>
<td>1100</td>
<td>[1, 1, 0, 0, 0, 0]^T</td>
<td>1100</td>
<td>[1, 0, 1, 0, 0, 1]^T</td>
</tr>
<tr>
<td>1101</td>
<td>[1, 1, 0, 0, 0, 0]^T</td>
<td>1101</td>
<td>[1, 0, 1, 0, 0, 1]^T</td>
</tr>
<tr>
<td>1110</td>
<td>[1, 1, 0, 0, 0, 0]^T</td>
<td>1110</td>
<td>[1, 0, 1, 0, 0, 1]^T</td>
</tr>
<tr>
<td>1111</td>
<td>[1, 1, 0, 0, 0, 0]^T</td>
<td>1111</td>
<td>[1, 0, 1, 0, 0, 1]^T</td>
</tr>
</tbody>
</table>
employs \( n_t = 4 \) and the nonhierarchical GSSK employs \( n_t = 3 \) in this example.

4.3. Example C

We consider the system with \( N_T = 12 \). This larger system affords more design flexibility. We consider 1 or 2 HP bits. For \( C^{(\text{HP})} = 1 \), the first-level design determines the two symbols ("black points") as \([0, 1, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0]^{T}\) and \([1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0]^{T}\), achieving the optimal \( d^{(\text{HP})} = 12 \). The second-level design determines the sphere radius as \( r = 4 \). Among the 36 symbols on the sphere of radius 2, 225 symbols on the sphere of radius 4, and the center of the sphere, 256 symbols are arbitrarily selected to yield \( C^{(\text{LP})} = 8 \). Both sphere and Voronoi graph methods yield the same LP capacity. For \( C^{(\text{HP})} = 2 \), the first-level design determines the four symbols ("black points") as \([1, 1, 1, 0, 0, 0, 1, 1, 0, 1, 0, 0]^{T}\), \([1, 0, 0, 1, 1, 0, 1, 0, 1, 0, 1, 0]^{T}\), \([0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1]^{T}\), and \([0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 1, 1]^{T}\), achieving the optimal pairwise distance \( d^{(\text{HP})} = 8 \). If the sphere method is adopted for the second-level design, the sphere radius is determined as \( r = 2 \). Among the 36 symbols on the sphere of radius 2 and the center of the sphere, 32 symbols are arbitrarily selected to yield \( C^{(\text{LP})} = 5 \). If the Voronoi graph method is adopted for the second-level design, among the 157 symbols in each Voronoi cell, 128 symbols are arbitrarily selected to yield \( C^{(\text{LP})} = 7 \). As can be seen in this example, there may exist symbols in a Voronoi cell but not inside the sphere of radius \( r \). As a result, the Voronoi graph method may achieve an increased LP capacity as compared to the sphere method. Both H-GSSK (2 HP, 7 LP) and H-GSSK (1 HP, 8 LP) employ \( n_t = 6 \) and the nonhierarchical GSSK employs \( n_t = 5 \) in achieving the same sum rate of 9 bits/s/Hz in this example.

5. Performance results

Here, we demonstrate the bit-error-rate (BER) versus \( E_b/N_0 \) performance of the proposed H-GSSK schemes, where \( E_b \) represents the average energy of a single bit and is given by \( E_b = \mathbb{E}[|\hat{x}|^2]/m \) for \( m \) bits/s/Hz transmission. Symmetric systems \((N_T = N_R)\) are considered, although this is not a constraint for the proposed method. ML detection is adopted for all schemes.

Fig. 4 shows the performance results for Example A in Section 4. As can be seen, the HP performance of H-GSSK is slightly better than its LP performance which is identical to the performance of the nonhierarchical GSSK. Interestingly, unlike conventional PSK/PAM/QAM, for GSSK modulation increasing the HP protection does not necessarily force a compromise on the LP protection given the same average symbol power.

Fig. 5 shows the performance results for Example B. Compared with the nonhierarchical GSSK, H-GSSK achieves enhanced HP protection at the compromise of LP protection. In addition, H-GSSK outperforms 32-HQAM (with gray-coded bit mapping) given the same average symbol power and the same rate, at the cost of more required transmitter RF chains.

Fig. 6 shows the performance results for Example C. We compare schemes with different rates after equalizing the bit energy. With the same \( C^{(\text{HP})} = 2 \), H-GSSK (2 HP, 7 LP) shows inferior HP performance but slightly superior LP performance than H-GSSK (2 HP, 5 LP). H-GSSK (2 HP, 7 LP) observes inferior HP performance because the Voronoi graph method selects “white points” that are closer to the boundaries of the Voronoi cells as compared to the sphere method, resulting in the decreased minimum distance between symbols addressed by the HP bits. On the other hand, selecting “white points” that are more widely scattered in the hyperspace in the Voronoi graph method enlarges the average pairwise distances between symbols addressed by the LP bits, thus resulting in the slightly better LP performance for H-GSSK (2 HP, 7 LP). At the same sum rate, H-GSSK (1 HP, 8 LP) exhibits remarkably better HP performance than H-GSSK (2 HP, 7 LP), with nearly identical LP performance. The larger \( d^{(\text{HP})} \) and consequently the larger minimum distance between symbols addressed by the HP bits lead to the better HP performance.
performance of H-GSSK (1 HP, 8 LP). The comparable pairwise distances between symbols addressed by the LP bits lead to the comparable LP performance for both schemes.

6. Conclusion

A systematic approach to designing hierarchical GSSK modulation for two-level UEP has been described. Set partitioning in the multidimensional space in the first level and graph-based methods to select constellation points inside each partitioned subset in the second level were proposed. Simulation showed that for GSSK modulation with some parameter $n$, increasing the capacity of the HP bits decreases the HP performance, and increasing the capacity of the LP bits decreases the HP performance and slightly increases the LP performance. Extension of the proposed UEP design method to space–time SSK schemes will be a worthwhile future work.

References


