Lower Bounds on the Correlation Property for OFDM Sequences with Spectral-Null Constraints

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***Abstract*—Sequences with specific autocorrelation (AC) and cross-correlation (CC) properties are crucial components in radar and wireless communications. In this paper, we derive the theoretical bounds on the AC and CC for OFDM sequences with constraints of spectral nulls, e.g., the mandatory nulls on the DC sub-carrier and guardbands in OFDM systems. The bounds and trade-off limits are provided for the properties of sequences, including the peak AC and CC levels, the cardinality of the sequence set, the sequence length, and the temporal length of the low correlation zone. We also investigate the trade-offs of correlations among *sequence sets*. The presented trade-off limits can serve as guidelines for applications where the performance measures or design criteria are related to the peak AC and CC levels.**

***Index Terms*—Null-subcarrier, autocorrelation, cross-correlation, complementary sequences, zero correlation zone, low correlation zone.**

I. Introduction

In many communication and radar applications, it is desired to design a set of sequences whose autocorrelation (AC) function is impulse-like and whose cross-correlation (CC) function is low or zero at all time delays. Such sequences are used to perform fundamental communication functionalities, e.g., synchronization, frequency offset estimation, and channel estimation. In these applications, the impulse-like AC property avoids self-interference caused by multi-paths, while the zero CC property prevents the interference from other co-channel users or antennas. However, as proved by Welch and Sarwate [1], [2], the AC and CC properties for an arbitrary set of sequences are constrained; there are trade-offs among the maximum correlation, sequence length, and the cardinality of the sequence set. Based on the constraints, the impulse-like AC and zero CC at all temporal shifts for a set of sequences are not simultaneously achievable.

In recent years, families of sequences referred to as zero-correlation zone (ZCZ) sequences and low-correlation zone (LCZ) sequences have attracted numerous research interests. Without violating the Welch bound, these sequences exhibit zero correlation or low correlation for all time delays within a desired temporal window. The sequences have been investigated for applications, including the spreading sequences for quasi-synchronous code division multiple access (QS-CDMA) systems [3], [4], and the training sequences for single-input single-output (SISO) and multi-input multi-output (MIMO) systems [5]–[11]. In [5], it was shown that training sequences with the impulse-like AC property minimize the average Cramér-Rao lower bound (CRLB) for the carrier frequency offset (CFO) estimation in SISO frequency-selective Rayleigh fading channels. The other work [6] investigated the CFO estimation for MIMO cases and showed that the ZCZ sequences minimize a channel-independent CRLB. For the application of channel estimation, the previous studies [7]–[11] have shown that the ZCZ sequences which have zero AC (excluding a peak at zero correlation lag) and zero CC for all lags within the maximum channel delay spread are the optimal training signals to minimize the estimation variance.

Lower bounds and trade-offs for the parameters of the ZCZ/LCZ sequence families are derived in [12]–[14], and are also summarized in [15]. Such bounds show the fundamental limits among the sequence parameters and serve as guidelines for sequence design in applications. For example, to obtain a set of M ZCZ sequences with length L, the bounds show that the size of the zero-correlation zone, Z_{cz}, must be no greater than $\frac{L}{M}$. In the application using the ZCZ sequences as the training sequences over an M-by-N MIMO channel with maximum channel delay spread equal to $\tau_{\max}$, it has been shown that the least-squares channel estimator achieves the minimum possible CRLB if $\tau_{\max} < Z_{\text{cz}}$. To achieve the CRLB, the ZCZ sequences must be chosen with parameters satisfying $\frac{L}{M} > \tau_{\max}$. In addition to the application for the case of ZCZ sequences, the trade-off bounds are applicable for various applications whose performance measures or design criteria are related to the peak AC and CC levels.

Motivated by the training sequence design in MIMO systems, in this work we aim to find out the trade-offs for the parameters of the ZCZ/LCZ sequence families when spectral-null constraints are taken into consideration. In OFDM systems, certain sub-carriers are reserved and are prohibited to transmit signals [16]. For example, the DC sub-carrier is reserved, i.e., spectrally nulled, to avoid offsets in the D/A and A/D converter in RF systems. In addition, the guardbands at spectrum edges are reserved to prevent interferences on adjacent frequency channels. Figure 1 illustrates the arrangement of null sub-carriers in IEEE 802.11a standard. The previous

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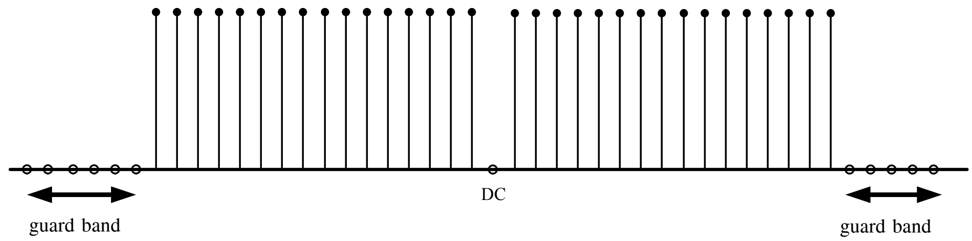

Fig. 1. Null sub-carriers in IEEE 802.11a standard.

investigations on AC and CC properties on generic sequences are not directly applicable to such OFDM systems due to the constraints on OFDM spectral nulls. While the past research works on AC and CC properties have substantially contributed to the design and theoretical understanding of the sequences, however, the studies on the sequences with constraints of spectral nulls are rare. As a direct solution for obtaining sequences satisfying the spectral-null constraints, it is feasible to filter the existing bound-achieving sequences through a spectrum mask that specifies the null constraints [8]. However, since the original bound-achieving sequences are not optimized under the explicit constraints of the spectral nulls, the filtered sequences may not exhibit correlation properties as well as the original sequences do. An intriguing question that then follows is what the trade-off limit of the spectrum-constrained sequence set is. Whether the filtered sequences achieve this theoretical limit also remains to be answered. To the best of our knowledge, this is the first work that investigates the lower bounds and trade-offs for the maximum AC and CC of the OFDM sequences with spectral-null constraints. The results for both periodic and aperiodic correlation functions will be presented.

Furthermore, we investigate the trade-off limit for the AC and CC properties among *sequence sets*. Each set consists of K sequences transmitted over K parallel channels. The well-known Golay complementary sets [17] are special cases with $K = 2$ and zero aperiodic AC for all non-zero lags. In [18], the authors proposed a *two-sided* pilot structure and applied one complementary sequence set as the training signal for SISO channel estimation. They suggested a criterion based on the AC of the pilot sequences and showed that the complementary sequence set minimizes both the design criterion and the CRLB of the estimation variance. Extending to the MIMO case, the work [19] utilized multiple uncorrelated Golay complementary sets of polyphase sequences to estimate frequency-selective MIMO channels. We will present the trade-off among the sequence parameters and correlation properties given that each transmitted sequence must follow the spectral-null constraints.

The rest of this paper is organized as follows. In Section II, we define notations and review the properties that are used in the subsequent discourse. Section III presents our main results on the trade-offs for the spectrally constrained sequences for both the periodic and aperiodic correlations. In Section IV, we investigate the bounds and tradeoffs for a group of sequence sets. In Section V, we demonstrate families of sequences which achieve the derived bounds. The concluding remarks are given in Section VI.

II. Preliminaries

As introduced in Section I, two classes of correlation properties are investigated in this work:

- Correlation among the sequences in a sequence set;
- Correlation among sequence sets.

In this section, we present fundamental definitions and properties for the correlation among sequences within a sequence set, by mostly following the conventions in [1] and [11]. The mathematical definition for the correlation among sequence sets will not be given until Section IV, to avoid the confusion between the two classes of correlation properties.

In this paper, vectors and matrices representing sequences and sequence sets are written in boldface with matrices in capitals, and all vectors are column vectors. The sequences and their elements discussed in this paper are complex-valued. The Euclidean norm of each sequence is normalized to 1. The Hermitian and transpose operators over matrices or vectors are denoted by $(\cdot)^H$ and $(\cdot)^T$, respectively.

Definition 1: (Periodic CC) Let $(n)_L$ denote n (mod L). The periodic CC function of two length-L sequences $\mathbf{u} = [u(0), \ldots, u(L-1)]^T$ and $\mathbf{v} = [v(0), \ldots, v(L-1)]^T$ is defined by

$$\theta_{\mathbf{u},\mathbf{v}}(\tau) = \sum_{i=0}^{L-1} u((i+\tau)_L)v^*(i),\ \tau = 0, \ldots, L-1. \quad (1)$$

The periodic AC function of $\mathbf{u}$ is $\theta_{\mathbf{u},\mathbf{u}}(\tau)$. ■

Definition 2: (Aperiodic CC) The aperiodic CC function of two length-L sequences $\mathbf{u}$ and $\mathbf{v}$ is defined by

$$\hat{\theta}_{\mathbf{u},\mathbf{v}}(\tau) = \begin{cases} \sum\limits_{i=0}^{L-\tau-1} u(i+\tau)v^*(i), & \tau = 0, \ldots, L-1; \\ \sum\limits_{i=0}^{L+\tau-1} u(i)v^*(i-\tau), & \tau = -L+1, \ldots, -1. \end{cases} \quad (2)$$

■

Definition 3: (Low/zero correlation zone sequence set) Let $\mathcal{S}$ denote a set of M complex-valued sequences of length L. We consider the integer time delay τ within the maximum range of interest L_{cz}, i.e., $|\tau| < L_{\text{cz}}$. The maximum magnitude of the AC function, denoted by θ_a, and the maximum magnitude of the CC function, denoted by θ_c, are defined as

$$\theta_a = \max_{\mathbf{u}\in\mathcal{S}; 0<|\tau|<L_{\text{cz}}} |\theta_{\mathbf{u},\mathbf{u}}(\tau)|; \quad (3)$$

$$\theta_c = \max_{\mathbf{u},\mathbf{v}\in\mathcal{S}; \mathbf{u}\neq\mathbf{v}; |\tau|<L_{\text{cz}}} |\theta_{\mathbf{u},\mathbf{v}}(\tau)|. \quad (4)$$

The sequence set $\mathcal{S}$ is then said to be a *low correlation zone (LCZ)* sequence set with parameters $(M, L, L_{\text{cz}}, \theta_a, \theta_c)$. The interval $|\tau| < L_{\text{cz}}$ is called the *low correlation zone*, and L_{cz}

is referred to as the size of the low correlation zone. For the special case with $\theta_a = \theta_c = 0$, the sequence set $\mathcal{S}$ is said to be a *zero correlation zone (ZCZ)* sequence set with parameters $(M, L, Z_{\mathrm{cz}}, 0, 0)$, where $Z_{\mathrm{cz}} \equiv L_{\mathrm{cz}}$. ■

Similar to Definition 3, for aperiodic correlation $\hat{\theta}_{\mathbf{u},\mathbf{v}}(\tau)$, given the delay interval $|\tau| < L_{\mathrm{cz}}$, we have the following corresponding parameters: $\hat{\theta}_a$ and $\hat{\theta}_c$.

The following two definitions are related to the spectral-null constrained OFDM sequences, i.e., the main subjects in this work.

Definition 4: (Discrete Fourier Transform (DFT), Inverse-DFT (IDFT), and OFDM sequences) We suppose that the sub-carriers in an OFDM system are indexed by $0, 1, \ldots, L-1$. An OFDM sequence $\mathbf{u} = [u(0), \ldots, u(L-1)]^T$ is synthesized by taking L-point IDFT on a length-L frequency-domain sequence $\mathbf{u}_{\mathrm{f}} = [U(0), \ldots, U(L-1)]$:

$$u(n) = \frac{1}{L}\sum_{k=0}^{L-1} U(k) e^{j2\pi\frac{kn}{L}}, \tag{5}$$

where $U(k)$ is the symbol carried by the k-th sub-carrier. We define $\mathbf{F}$ as the linear transformation matrix such that $\mathbf{u} = \mathbf{F}^{-1}\mathbf{u}_{\mathrm{f}}$. The matrix $\mathbf{F}$ is referred to as the DFT matrix, and the vector $\mathbf{u}_{\mathrm{f}}$ is referred to as the L-point DFT of $\mathbf{u}$. ■

Definition 5: (Spectral-null constraints) Following Definition 4, we define a set Ω to specify spectral-null constraints. The set Ω consists of all the indexes of the sub-carriers allowed to be active. The set Ω is a subset of $\{0, \ldots, L-1\}$ and is of cardinality $|\Omega|$. The complement set $\Omega' \equiv \{0, \ldots, L-1\} \setminus \Omega$ specifies the indexes of the sub-carriers that must be nulled. In other words, for a length-L OFDM sequence $\mathbf{u}$, $U(k)$ is zero for all $k \in \Omega'$. ■

The following property and definitions were introduced in [1] and will be used in later sections.

Definition 6: Given $\mathcal{S}$, a set of sequences of the same length and the same Euclidean norm of 1, we define $B_k(\mathcal{S})$ as

$$B_k(\mathcal{S}) \equiv \sum_{\mathbf{u},\mathbf{v}\in\mathcal{S}} |\mathbf{u}^H\mathbf{v}|^{2k}. \tag{6}$$

■

Property 1: Consider a set of length-L sequences $\mathcal{S}$. The cardinality of the set is $|\mathcal{S}|$. Welch showed that the sum of the inner products of any $\mathbf{u}, \mathbf{v}$ pairs in $\mathcal{S}$ follows the inequality below [1]:

$$B_k(\mathcal{S}) \geq \frac{|\mathcal{S}|^2}{\binom{L+k-1}{k}}, \tag{7}$$

where k can be any positive integer.

Definition 7: We define D^{-1} as the *delay operator* that cyclically shifts down the components of a vector by one place and $D^{-i} = \underbrace{D^{-1} \cdot D^{-1} \cdots D^{-1}}_{i-fold}$ as the operator that shifts down the components of a vector cyclically by i places. ■

III. Bounds for OFDM Sequences with Spectral-null Constraints

In this section, we consider the trade-off among the correlation properties and sequence parameters on a set of OFDM sequences satisfying the same spectral-null constraints. To facilitate the subsequent discussion, we let $\mathcal{S}_\Omega$ denote a set of length-L OFDM sequences satisfying the same spectral-null constraints specified by Ω. The cardinality of Ω is $|\Omega| \leq L$.

In the following, we first generalize the inequality in Property 1 to the spectrally constrained OFDM sequences. Utilizing this generalized inequality, we next derive the correlation trade-off when the periodic correlation or the aperiodic correlation is considered.

A. Sum of inner products inequality for spectrally constrained sequences

The following lemma generalizes Property 1 to the case with spectral-null constraints.

Lemma 1: (Sum of inner products inequality)

Given Ω and any OFDM-sequence set $\mathcal{S}_\Omega$ satisfying the null constraints specified by Ω, we have

$$B_k(\mathcal{S}_\Omega) \geq \frac{|\mathcal{S}_\Omega|^2}{\binom{|\Omega|+k-1}{k}}, \tag{8}$$

where k can be any positive integer.

Proof: From Definition 6, we have

$$B_k(\mathcal{S}_\Omega) = \sum_{\mathbf{u},\mathbf{v}\in\mathcal{S}_\Omega} |\mathbf{u}^H\mathbf{v}|^{2k} \tag{9}$$

$$= \sum_{\mathbf{u},\mathbf{v}\in\mathcal{S}_\Omega} |(\frac{1}{\sqrt{L}}\mathbf{F}\mathbf{u})^H(\frac{1}{\sqrt{L}}\mathbf{F}\mathbf{v})|^{2k} \tag{10}$$

$$= \sum_{\mathbf{u},\mathbf{v}\in\mathcal{S}_\Omega} |\mathbf{u}_{\mathrm{d}}^H\mathbf{v}_{\mathrm{d}}|^{2k} \geq \frac{|\mathcal{S}_\Omega|^2}{\binom{|\Omega|+k-1}{k}}, \tag{11}$$

where the DFT matrix $\mathbf{F}$ is a scaled unitary matrix, and the unit-norm vectors $\mathbf{u}_{\mathrm{d}}$ and $\mathbf{v}_{\mathrm{d}}$ are the $|\Omega|$-by-1 subvectors that lie in the rows of $\frac{1}{\sqrt{L}}\mathbf{F}\mathbf{u}$ and $\frac{1}{\sqrt{L}}\mathbf{F}\mathbf{v}$ indexed by Ω. The derivation of (10) utilizes the property that the linear transformation of $\frac{1}{\sqrt{L}}\mathbf{F}$ is inner-product preserving. The inequality (11) results from Property 1. ■

Hereafter we refer to (8) as *sum of inner products inequality*. Property 1 can be treated as a special case of (8) with $|\Omega| = L$, i.e., all sub-carriers are allowed to be active.

B. Periodic correlation bounds

Here we consider the periodic correlation properties among the sequences in $\mathcal{S}_\Omega$. The sequence parameters and the sequence correlation properties must obey the theorem below.

Theorem 1: (Generalized periodic correlation bounds)

The set $\mathcal{S}_\Omega$ can be treated as an LCZ sequence set with parameters $(M, L, L_{\mathrm{cz}}, \theta_a, \theta_c)$ and $M = |\mathcal{S}_\Omega|$. These parameters must satisfy the following inequality:

$$p_a \cdot \theta_a^{2k} + p_c \cdot \theta_c^{2k} + p_s \geq \frac{(ML_{\mathrm{cz}})^2}{\binom{|\Omega|+k-1}{k}}, \tag{12}$$

where

$$\begin{aligned} p_a &= ML_{\mathrm{cz}}(L_{\mathrm{cz}}-1), \\ p_c &= ML_{\mathrm{cz}}(ML_{\mathrm{cz}}-L_{\mathrm{cz}}), \\ p_s &= ML_{\mathrm{cz}}, \end{aligned} \tag{13}$$

and k can be any positive integer. ■

Proof: Similar to the approach in [1], we consider a set $\mathcal{E}_\Omega \equiv \{D^i\mathbf{u} \mid \forall\mathbf{u} \in \mathcal{S}_\Omega,\ \forall i = 0, \dots, L_{\text{cz}} - 1\}$, which is of cardinality $|\mathcal{E}_\Omega| = ML_{\text{cz}}$. The periodic AC and CC values for sequences in $\mathcal{S}_\Omega$ with $|\tau| < L_{\text{cz}}$ are the inner products of the sequence pairs from the set $\mathcal{E}_\Omega$ [1].

This theorem is proved by deriving the lower and upper bounds of $B_k(\mathcal{E}_\Omega)$. By applying the sum of inner-products inequality for spectrally constrained sequences in (8) to the set $\mathcal{E}_\Omega$, we have

$$B_k(\mathcal{E}_\Omega) \geq \frac{(ML_{\text{cz}})^2}{\binom{|\Omega|+k-1}{k}}. \tag{14}$$

We note that (8) is applicable to $\mathcal{E}_\Omega$ because the DFT of the sequence $D^i\mathbf{u}$ must meet the spectral-null constraints specified by Ω, irrespective of the phase rotation in the frequency domain induced by the time-shifting operator, D^i [20]. The upper bound of $B_k(\mathcal{E}_\Omega)$ can be derived as follows. We categorize the pairs $(\mathbf{x}, \mathbf{y})$ for all $\mathbf{x}, \mathbf{y} \in \mathcal{E}_\Omega$ into three sets:

$$\begin{aligned} \mathcal{P}_s &= \{(D^i\mathbf{u}, D^j\mathbf{v}) | \forall \mathbf{u}, \mathbf{v} \in \mathcal{S}_\Omega;\ \mathbf{u} = \mathbf{v}; \\ &\quad \forall\ i = j,\ i = 0, \dots, L_{\text{cz}} - 1\}; && (15) \\ \mathcal{P}_a &= \{(D^i\mathbf{u}, D^j\mathbf{v}) | \forall \mathbf{u}, \mathbf{v} \in \mathcal{S}_\Omega;\ \mathbf{u} = \mathbf{v}; \\ &\quad \forall\ i, j = 0, \dots, L_{\text{cz}} - 1;\ i \neq j\}; && (16) \\ \mathcal{P}_c &= \{(D^i\mathbf{u}, D^j\mathbf{v}) | \forall \mathbf{u}, \mathbf{v} \in \mathcal{S}_\Omega;\ \mathbf{u} \neq \mathbf{v}; \\ &\quad \forall\ i, j = 0, \dots, L_{\text{cz}} - 1\}. && (17) \end{aligned}$$

The cardinalities of $\mathcal{P}_s$, $\mathcal{P}_a$, and $\mathcal{P}_c$ are denoted by p_s, p_a, and p_c, respectively. As a result, the sum $B_k(\mathcal{E}_\Omega)$ is upper bounded by

$$p_a \cdot \theta_a^{2k} + p_c \cdot \theta_c^{2k} + p_s \geq B_k(\mathcal{E}_\Omega). \tag{18}$$

Combining (14) and (18), the proof is completed. ■

Corollary 1: (Generalized bounds for ZCZ seqences)

For the case with $\theta_a = \theta_c = 0$, (12) becomes

$$\binom{|\Omega| + k - 1}{k} \geq ML_{\text{cz}}. \tag{19}$$

The trade-off bound with $k = 1$ is the tightest one among all k's due to $\binom{|\Omega|+k-1}{k} \geq |\Omega|$ for all positive integers k and $|\Omega|$. The bound $|\Omega| \geq ML_{\text{cz}}$ implies that, to obtain a set of M ZCZ sequences with a ZCZ size of L_{cz}, the number of active sub-carriers should be no less than ML_{cz}. ■

The following corollaries further show that the previous bounds in existing literature for spectrum-unconstrained sequences are special cases of the bounds in Theorem 1.

Corollary 2: (Tang-Fan-Matsufuji bound)

For the special case with $|\Omega|$=L, i.e., all sub-carriers can be active, the inequality in (12) leads to the Tang-Fan-Matsufuji bound presented in [12]. ■

Corollary 3: (Welch bound and Sarwate bound)

For the special case with $|\Omega|$=L and L_{cz}=L, the inequality in (12) leads to the Welch bound [1] and Sarwate bound [2]. ■

C. Aperiodic correlation bounds

Before presenting the aperiodic correlation bounds for the spectral-constrained sequences, we briefly review the correlation bounds for the case without spectral-null constraints. We define $\tilde{\mathbf{z}}(\mathbf{u}) = [u(0), \dots, u(L-1), \mathbf{0}_{1\times(L_{\text{cz}}-1)}]^T$ as a vector padded by $L_{\text{cz}} - 1$ zeros. The length of $\mathbf{z}$ is $L + L_{\text{cz}} - 1$. Consider a set $\hat{\mathcal{E}} \equiv \{D^i\tilde{\mathbf{z}}(\mathbf{u}) \mid \forall\mathbf{u} \in \mathcal{S},\ i = 0, \dots, L_{\text{cz}} - 1\}$. The aperiodic AC and CC values for sequences in $\mathcal{S}$ with $|\tau| < L_{\text{cz}}$ are the inner products of the sequence pairs from the set $\hat{\mathcal{E}}$ [1]. By applying Property 1 to the set $\hat{\mathcal{E}}$, we have

$$\begin{aligned} B_k(\hat{\mathcal{E}}) &\geq \frac{(ML_{\text{cz}})^2}{\binom{L+(L_{\text{cz}}-1)+k-1}{k}}; && (20) \\ B_k(\hat{\mathcal{E}}) &\leq p_a \cdot \hat{\theta}_a^{2k} + p_c \cdot \hat{\theta}_c^{2k} + p_s && (21) \\ &\leq (p_a + p_c)\hat{\theta}_{\max}^{2k} + p_s, && (22) \end{aligned}$$

where p_a, p_c, and p_s are given in (13), and $\hat{\theta}_{\max} = \max\{\hat{\theta}_a, \hat{\theta}_c\}$. Combining the inequalities (20) and (22) leads to the aperiodic correlation bound for LCZ sequences presented in [13].

In contrast to the above derivation, to further take the spectral-null constraints into account, we intend to construct a $2L$-by-1 vector $\mathbf{z}(\mathbf{u})$ as:

$$\mathbf{z}(\mathbf{u}) = [u_0, \dots, u_{L-1}, \mathbf{0}_{1\times L}]^T, \tag{23}$$

where L zeros are padded. Consider a set $\hat{\mathcal{E}}_\Omega \equiv \{D^i\mathbf{z}(\mathbf{u}) \mid \forall\mathbf{u} \in \mathcal{S}_\Omega,\ i = 0, \dots, L_{\text{cz}} - 1\}$. The AC and CC values for $|\tau| < L_{\text{cz}}$ are the inner products of the sequence pairs from the set $\hat{\mathcal{E}}_\Omega$. The m-th element of the $2L$-point DFT of $\mathbf{z}(\mathbf{u})$ can be expressed as

$$Z_{\mathbf{u}}(m) = \sum_{n=0}^{L-1} u(n)e^{-j2\pi\frac{mn}{2L}}. \tag{24}$$

It is clear that

$$Z_{\mathbf{u}}(2m) = \sum_{n=0}^{L-1} u(n)e^{-j2\pi\frac{mn}{L}}, \tag{25}$$

which is the L-point DFT of the vector $\mathbf{u}$. Therefore, the $2L$-point DFT of any sequence in $\hat{\mathcal{E}}_\Omega$ has at least $L - |\Omega|$ zeros at the indexes specified by $2m$ for all $m \in \Omega'$. Consequently, the sum of inner product inequality (8) can be directly applied to $\hat{\mathcal{E}}_\Omega$, and we have

$$B_k(\hat{\mathcal{E}}_\Omega) \geq \frac{(ML_{\text{cz}})^2}{\binom{|\Omega|+L+k-1}{k}}. \tag{26}$$

Compared to the lower bound in (20), which was derived without considering the spectral nulls, the lower bound in (26) is tighter for $|\Omega| < (L_{\text{cz}} - 1)$. On the other hand, the upper bound of $B_k(\hat{\mathcal{E}}_\Omega)$ is given by

$$B_k(\hat{\mathcal{E}}_\Omega) \leq p_a \cdot \hat{\theta}_a^{2k} + p_c \cdot \hat{\theta}_c^{2k} + p_s. \tag{27}$$

By combining (20), (26), and (27), we have the following generalized bounds on the aperiodic correlation properties for spectrally constrained sequences.

Theorem 2: (Generalized aperiodic correlation bounds)

The set $\mathcal{S}_\Omega$ can be treated as an LCZ sequence set with sequence parameters $(M, L, L_{\text{cz}}, \hat{\theta}_a, \hat{\theta}_c)$ and $M = |\mathcal{S}_\Omega|$. These parameters must satisfy the following inequality:

$$p_a \cdot \hat{\theta}_a^{2k} + p_c \cdot \hat{\theta}_c^{2k} + p_s \geq \frac{(ML_{\text{cz}})^2}{\min\{\binom{|\Omega|+L+k-1}{k}, \binom{L+L_{\text{cz}}+k-2}{k}\}},$$

where p_a, p_c, and p_s are given in (13), and k can be any positive integer.

The relationship between the previous bounds in [1], [2], [13] and our generalized bounds for the aperiodic correlation is similar to that stated in corollaries 2 and 3 for the periodic case.

IV. Correlation Bounds for Sequence Sets Transmitted over Parallel Channels

In [1], Welch had studied the theoretical bounds on the aperiodic correlation properties of sequence sets transmitted over multi-channels. Suppose K sequences are transmitted through K separate channels. Each sequence is of length L. We denote the set of the K sequences by a K-by-L matrix, $\mathbf{U}$, whose i-th row vector is the i-th sequence. The receiving terminal matches these signals by the sequences of another sequence set $\mathbf{V}$ and sums the matched outputs. The sum is said to be the correlation of the two sets $\mathbf{U}$ and $\mathbf{V}$. This concept was first introduced by Golay [17] for $K = 2$ and was generalized to the cases with $K > 2$ in [21]. More precisely, the aperiodic correlation between two sets, $\mathbf{U} = [\mathbf{u}_0, \mathbf{u}_1, \ldots, \mathbf{u}_{K-1}]^T$ and $\mathbf{V} = [\mathbf{v}_0, \mathbf{v}_1, \ldots, \mathbf{v}_{K-1}]^T$, is defined as

$$\hat{\Theta}_{\mathbf{U},\mathbf{V}}(\tau) = \sum_{i=0}^{K-1} \hat{\theta}_{\mathbf{u}_i\mathbf{v}_i}(\tau). \quad (28)$$

The periodic correlation between two sets $\mathbf{U}$ and $\mathbf{V}$ is defined as

$$\Theta_{\mathbf{U},\mathbf{V}}(\tau) = \sum_{i=0}^{K-1} \theta_{\mathbf{u}_i\mathbf{v}_i}(\tau). \quad (29)$$

In this section, we aim to find the trade-off limit for the multi-channel periodic/aperiodic correlation properties under the spectral-null constraints. We let Ω indicate the specific sub-carriers allowed to be active, and every sequence follows the same spectral-null constraints. Suppose there are M spectrum-constrained sequence sets. Each set, represented by a K-by-L matrix, contains K sequences. All the sequences are of length L and of Euclidean-norm of 1. We define a super set $\mathcal{S}_\Omega^{\text{mul}}$ consisting of the M sequence sets. Considering the integer time delay τ within the maximum range of interest L_{cz}, i.e., $|\tau| < L_{\text{cz}}$, we define the maximum magnitude of the periodic AC function, denoted by Θ_a, and the maximum magnitude of the periodic CC function, denoted by Θ_c, as

$$\Theta_a = \max_{\mathbf{U}\in\mathcal{S}_\Omega^{\text{mul}};0<|\tau|<L_{\text{cz}}} |\Theta_{\mathbf{U},\mathbf{U}}(\tau)|; \quad (30)$$

$$\Theta_c = \max_{\mathbf{U},\mathbf{V}\in\mathcal{S};\mathbf{U}\neq\mathbf{V};|\tau|<L_{\text{cz}}} |\theta_{\mathbf{U},\mathbf{V}}(\tau)|. \quad (31)$$

Similarly, for aperiodic correlation $\hat{\Theta}_{\mathbf{U},\mathbf{V}}(\tau)$, given the delay interval $|\tau| < L_{\text{cz}}$, we have the corresponding parameters, $\hat{\Theta}_a$ and $\hat{\Theta}_c$. The next theorem presents the bounds for the correlation among the spectral-constrained sequence sets within $\mathcal{S}_\Omega^{\text{mul}}$.

Theorem 3: (Multi-channel correlation bounds)

For the super set $\mathcal{S}_\Omega^{\text{mul}}$ with cardinality $|\mathcal{S}_\Omega^{\text{mul}}| = M$, the inequalities below holds:

$$p_a \cdot \Theta_a^{2k} + p_c \cdot \Theta_c^{2k} + p_s \geq \frac{(ML_{\text{cz}})^2}{\binom{K|\Omega|+k-1}{k}}; \quad (32)$$

$$p_a \cdot \hat{\Theta}_a^{2k} + p_c \cdot \hat{\Theta}_c^{2k} + p_s \geq \frac{(ML_{\text{cz}})^2}{\min\{\binom{K(L+|\Omega|)+k-1}{k}, \binom{K(L+L_{\text{cz}}-1)+k-1}{k}\}}, \quad (33)$$

where p_a, p_c, and p_s are given in (13), and k can be any positive integer.

Proof: We first derive the bounds for the periodic case. In order to apply the sum inner product inequality for vectors to derive the trade-off limit, we follow Welch's method that vectorizes the matrices representing the sequence sets [1]. Given a sequence set $\mathbf{U}$, we define a vector $\mathbf{s}(\mathbf{U})$ formed by stacking the column vectors of $\mathbf{U}$ as

$$\mathbf{s}(\mathbf{U}) = [u_0(0)\cdots u_{K-1}(0),\ u_0(1)\cdots u_{K-1}(1), \cdots \\ \cdots,\ u_0(L-1)\ \cdots\ u_{K-1}(L-1)]^T. \quad (34)$$

From (34), the periodic CC between $\mathbf{U}$ and $\mathbf{V}$ can be expressed as

$$|\Theta_{\mathbf{U},\mathbf{V}}(\tau)| = |[\mathbf{s}(\mathbf{V})]^H[D^{\tau K}\mathbf{s}(\mathbf{U})]|. \quad (35)$$

Consider a set $\mathcal{E}_\Omega^{\text{mul}} \equiv \{D^{iK}\mathbf{s}(\mathbf{U}) \mid \forall \mathbf{U} \in \mathcal{S}_\Omega^{\text{mul}},\ i = 0,\ldots,L_{\text{cz}}-1\}$. The cardinality of $\mathcal{E}_\Omega^{\text{mul}}$ is $|\mathcal{E}_\Omega^{\text{mul}}| = ML_{\text{cz}}$. It then follows that the periodic AC and CC values among the sequence sets in $\mathcal{S}_\Omega^{\text{mul}}$ with $|\tau| < L_{\text{cz}}$ are the inner products of the sequence pairs in the set $\mathcal{E}_\Omega^{\text{mul}}$.

It can be proved that the KL-point DFT of each sequence in $\mathcal{E}_\Omega^{\text{mul}}$ has at least $K(L-|\Omega|)$ zeros at the indexes k's satisfying $(k)_L \in \Omega'$. The proof is detailed in Appendix. Applying the sum of inner product inequality to $\mathcal{E}_\Omega^{\text{mul}}$ and combining the upper bound and lower bound of $\mathcal{B}_k(\mathcal{E}_\Omega^{\text{mul}})$ lead to the result in (32).

Next we prove the bounds for the aperiodic case. We construct a $2KL$-by-1 vector $\mathbf{z}(\mathbf{U})$ as $[(\mathbf{s}(\mathbf{U}))^T, \mathbf{0}_{1\times KL}]^T$, where KL zeros are padded. Consider a set $\hat{\mathcal{E}}_\Omega^{\text{mul}} \equiv \{D^{iK}\mathbf{z}(\mathbf{U}) \mid \forall \mathbf{U} \in \mathcal{S}_\Omega^{\text{mul}},\ i = 0,\ldots,L_{\text{cz}}-1\}$. It then follows that the aperiodic AC and CC values for sequence sets in $\mathcal{S}_\Omega^{\text{mul}}$ with $|\tau| < L_{\text{cz}}$ are the inner products of the sequence pairs from the set $\hat{\mathcal{E}}_\Omega^{\text{mul}}$. It can also be proved that the $2KL$-point DFT of each sequence in $\hat{\mathcal{E}}_\Omega^{\text{mul}}$ has at least $K(L-|\Omega|)$ zeros at the indexes $2k$'s satisfying $(k)_L \in \Omega'$. We leave the proof also in Appendix. Applying the sum of inner product inequality (8) over $\hat{\mathcal{E}}_\Omega^{\text{mul}}$ and following the similar techniques presented in Section III-C complete the proof. ■

It can be easily verified that Theorem 1 and Theorem 2 arise as special cases in Theorem 3 with $K = 1$.

V. Examples of Bound-achieving Sequences

In this section, we present two families of ZCZ/LCZ sequences achieving the derived bounds under spectral-null constraints. The sequences are generated by filtering the periodic ZCZ sequences proposed in [11], which achieve the original Welch bound in [1] and have zero CC for all delays, through the spectral-null mask.

We first construct M ZCZ sequences, denoted by $\{\mathbf{x}_0, \ldots, \mathbf{x}_{M-1}\}$, in the frequency domain. Each of them is of

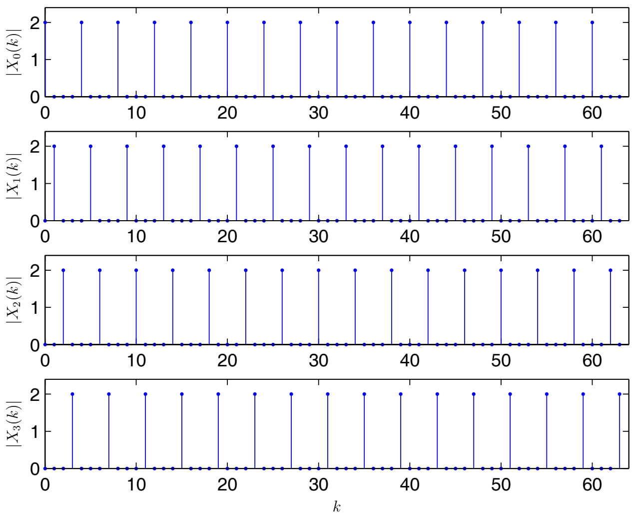

Fig. 2. ZCZ sequences in the frequency domain.

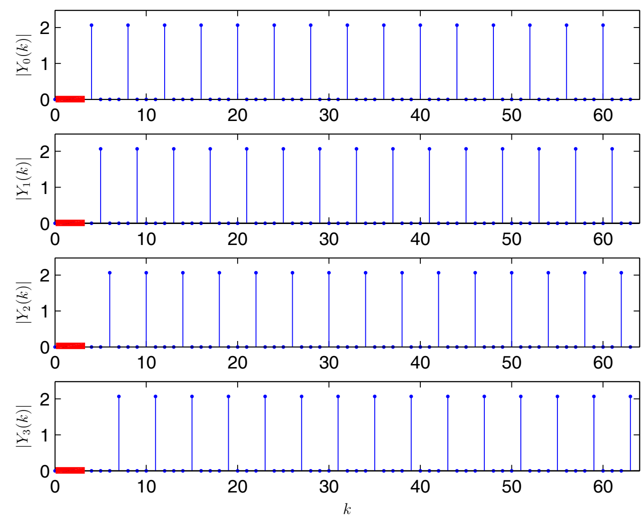

Fig. 3. Bound-achieving sequences in the frequency domain. The first M sub-carriers (with M=4) of each sequence are restricted to be nulled.

length $L = \alpha M$ with a positive integer α. The L-point DFT of $\mathbf{x}_i$, denoted by $[X_i(0), \ldots, X_i(k), \ldots, X_i(L-1)]^T$, is set according to the rule below [11]:

$$X_i(k) = \sqrt{\frac{L}{\alpha}} \cdot \sum_{w=0}^{\alpha-1} \delta(k - (Mw + i)), \tag{36}$$

where $\delta(\cdot)$ is the Kronecker delta function. For each sequence, equal power is assigned at equally-spaced sub-carriers with spacing M. Figure 2 illustrates these sequences in the frequency domain for the case of $M = 4$ and $\alpha = 16$. It can be proved that such sequences exhibit impulse-like periodic AC and zero periodic CC for all correlation lags within a zero-correlation zone of size α [11]. Based on this type of ZCZ sequences, we present two families of bound-achieving sequence sets under spectral-null constraints in the following.

A. Bound-achieving sequences constrained by equally spaced nulls

Following the construction of ZCZ sequences above, we note that in the scenario with α equally spaced spectral-nulls, only one of the generated ZCZ sequences violates the spectral-null constraints. The other $M-1$ sequences satisfying the constraints form a ZCZ sequence set with parameters $(M-1, L, Z_{\text{cz}}, 0, 0)$, where $Z_{\text{cz}} = \alpha$. On the other hand, given $\theta_a = \theta_c = 0$, the proposed tradeoff bound in Theorem 1 for a set of $M-1$ sequences becomes $Z_{\text{cz}} \le \frac{L-\alpha}{M-1} = \alpha$. Therefore, the derived bound further shows that the set of the $M-1$ sequences satisfying the spectral-null constraints is a bound-achieving ZCZ sequence set.

B. Bound-achieving sequences constrained by successive nulls

We consider another scenario with M successive spectral nulls. We will construct a set of M sequences, $\{\mathbf{y}_0, \ldots, \mathbf{y}_{M-1}\}$, having zero periodic cross-correlation. Each sequence is of length $L = ML_{\text{cz}}$. As will be shown, its maximum AC magnitude is minimized within an LCZ size L_{cz}.

Similar to Section V-A, we construct these sequences in the frequency domain. Let $\{(\sigma - M)_L, \ldots, (\sigma - 1)_L\}$ denote the set of null sub-carrier indexes, where σ can be arbitrary integer. The L-point DFT of $\mathbf{y}_i$, denoted by $[Y_i(0), \ldots, Y_i(k), \ldots, Y_i(L-1)]^T$, is given by:

$$Y_i(k) = \sqrt{\frac{L}{L_{\text{cz}} - 1}} \cdot \sum_{w=0}^{L_{\text{cz}}-2} \delta(k - (Mw + \sigma + i)_L). \tag{37}$$

Figure 3 illustrates the M sequences in the frequency domain. For each sequence $\mathbf{y}_i$, $L_{\text{cz}} - 1$ sub-carriers are made active and allocated with equal power. Following Parseval's theorem, we have

$$\sum_{n=0}^{L-1} |y_i(n)|^2 = \frac{1}{L} \sum_{k=0}^{L-1} |Y_i(k)|^2. \tag{38}$$

It can be verified that the generated sequences satisfy the basic assumption $|\mathbf{y}_i^H \mathbf{y}_i| = 1$.

The sequences generated by the rule in (37) are of zero periodic cross-correlation because $Y_i(k) Y_j^*(k)$ is zero for all k given $i \neq j$[11]. The magnitude of the periodic AC function $\theta_{\mathbf{y}_i,\mathbf{y}_i}(\tau)$ can be evaluated by

$$|\theta_{\mathbf{y}_i,\mathbf{y}_i}(\tau)| = \left| \frac{1}{L} \sum_{k=0}^{L-1} |Y_i(k)|^2 e^{j2\pi \frac{k\tau}{L}} \right| \tag{39}$$

$$= \frac{1}{L_{\text{cz}} - 1} \left| \sum_{k=0}^{L_{\text{cz}}-2} e^{j2\pi \frac{k\tau}{L_{\text{cz}}}} \right| \tag{40}$$

$$= \begin{cases} 1, & \text{if } \tau = mL_{\text{cz}},\ m \in \mathbb{Z}; \\ \frac{1}{L_{\text{cz}}-1}, & \text{otherwise.} \end{cases} \tag{41}$$

Equation (39) can be proved by the well-known fact $\theta_{\mathbf{y}_i,\mathbf{y}_i}(\tau) = y_i(\tau) \circledast y_i^*(-\tau)$, where $\circledast$ is a circular convolution operator. It can then be verified that such a set of sequences achieves the bound in (12) for $k = 1$, with $\theta_a = \frac{1}{L_{\text{cz}}-1}$ and $\theta_c = 0$.

We note that the general achievability of the proposed bounds is neither claimed nor proven in this paper. The sequence sets presented in this section are examples that achieve the proposed bounds. The existence and design of bound-achieving sequence sets with spectral constraints are open issues and left as future works.

C. Discussion on the performance of using bound-achieving sequences

At the end of this section, we rephrase the meaning of "bound-achieving" and discuss the performance of the spectrally constrained OFDM sequences meeting the proposed bound in practical applications. In general, the proposed bound for spectral-constrained LCZ sequences is a necessary condition for the *existence* of the LCZ sequences, in terms of the sequence parameters, maximum correlation levels, and the spectral constraints. Nevertheless, the "bound-achieving" characteristic does not directly imply the optimal performance in applications, as the optimality of performance is highly dependent on the criterion of interest in the applications.

Specially, for the applications investigated in [7]–[11], [18], [19], it has been proved that, to achieve the CRLB, the training sequences should be the ZCZ sequences whose zero-correlation zone is of size $Z_{\rm cz}$ greater than the maximum channel delay spread $\tau_{\rm max}$. Meanwhile, the proposed bound provides a necessary condition for the existence of the desired ZCZ sequences with $Z_{\rm cz} > \tau_{\rm max}$. If there are bound-achieving ZCZ sequences that satisfy the required spectral-null constraints and $Z_{\rm cz} > \tau_{\rm max}$, the sequences are the optimal training sequences in the sense of achieving the CRLB. Furthermore, the bound-achieving characteristic implies that if we add more spectral-null constraints, or request more ZCZ sequences or a larger zero-correlation zone, it is impossible to maintain the ZCZ property to achieve the CRLB.

On the other hand, the bound-achieving LCZ sequences, whose maximum AC or CC magnitude within the low-correlation zone is greater than zero, are not guaranteed to be optimal in the applications studied in [7]–[11], [18], [19]. Further proof is needed to verify whether or not these bound-achieving sequences can achieve the optimal performance, although the performance of these estimators may be reasonably good under the condition that the maximal AC and CC levels are quite small. The exact estimation variance can be calculated by the formula provided in [7]–[11], [18], [19].

VI. Conclusion and Future Work

In this paper, we provide bounds on the sequence correlations, the cardinality of the sequence set, the sequence length, and the temporal length of the low correlation zone, for OFDM sequences constrained by spectral nulls. The bounds on both periodic correlations and aperiodic correlations are presented. The presented bounds and trade-offs can serve as guidelines for sequence design in applications under spectral-null constraints. Exemplary sequences achieving the derived bounds are demonstrated. The open issues and future works include the general achievability of the derived bounds and the synthesis of sequence sets achieving or close to the bounds.

Appendix A
Zeros in the DFT of $\mathbf{s}(\mathbf{U})$ and $\mathbf{z}(\mathbf{U})$

We let $W_N = e^{j\frac{2\pi}{N}}$ be a primitive N-th root of unity where N is a positive integer. The KL-point DFT of $\mathbf{s}(\mathbf{U})$, denoted by $[S(0), S(1), \ldots, S(KL-1)]^T$, is computed by

$$\begin{aligned} S(k) &= \sum_{n=0}^{KL-1} s(n) W_{KL}^{-kn} && \text{(A.1)} \\ &= \sum_{n=0}^{L-1} u_0(n) W_{KL}^{-knK} + \sum_{n=0}^{L-1} u_1(n) W_{KL}^{-k(nK+1)} \\ &+ \cdots + \sum_{n=0}^{L-1} u_{K-1}(n) W_{KL}^{-k(nK+K-1)} && \text{(A.2)} \\ &= \sum_{m=0}^{K-1} W_{KL}^{-mk} \sum_{n=0}^{L-1} u_m(n) W_L^{-kn} && \text{(A.3)} \\ &= \sum_{m=0}^{K-1} W_{KL}^{-mk} U_m((k)_L), && \text{(A.4)} \end{aligned}$$

where $[U_m(0), \ldots, U_m(k), \ldots, U_m(L-1)]$ is the L-point DFT of $\mathbf{u}_m$. From (A.4) and the assumption that, for all m, $U_m(k)$ is zero for all $k \in \Omega'$, we conclude that $S((k)_L) = 0$ for all $(k)_L \in \Omega'$. Thus, the KL-point DFT of $\mathbf{s}(\mathbf{U})$ must have at least $K(L-|\Omega|)$ zeros, at the indexes k's satisfying $(k)_L \in \Omega'$.

For the vector $\mathbf{z}(\mathbf{U})$, a zero-padded version of $\mathbf{s}(\mathbf{U})$ with a doubled sequence length, its $2KL$-DFT must have at least $K(L-|\Omega|)$ zeros, at the indexes specified by $2k$ for all $(k)_L \in \Omega'$. We have proved this characteristic in (25) in Section III-C.

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