Iterative Source-Channel Decoding Design using Distortion based Index Assignment and Joint Redundant Information

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Abstract—This work presents a convolutional coded iterative source-channel decoding (ISCD) to improve the fidelity of joint source-channel coded systems through designing index assignment (IA). A proposed end-to-end distortion based IA generates the joint redundant bits to provide more useful information for source and channel decoders. Specifically, such joint redundant bit not only enhances the error-correcting capacity of source codes but also provides the self-concatenated convolutional codes instead of convolutional codes for ISCD. In contrast to the conventional ISCD, our proposed ISCD employs the self-iterative channel decoding algorithm, and the extrinsic information exchange can be further enhanced through joint redundant bits. With considering joint redundant bits, we propose a modified source decoder with the goal to extract the extrinsic information for the self-iterative channel decoder. Simulation results show the proposed ISCD with our designed IA can further improve the Parameter Signal-to-Noise ratio performance.

Index Terms—Source decoding, index assignment, iterative source-channel decoding.

I. INTRODUCTION

In [1]–[8], the iterative source-channel decoding (ISCD) is proven to significantly improve the fidelity of multimedia communications. The ISCD consisting of the soft-bit source decoding (SBSD) and the Soft-Input/Soft-Output (SISO) channel decoding refines the extrinsic information in turbo-like process. As a system uses non-redundant index assignment (IA) for the source codec parameters, the rate-1/2 convolutional encoder is employed to protect the output bits of the non-redundant IA. The IA plays a key factor to influence the performance of the ISCD schemes [4]. In [4], the extrinsic information transfer (EXIT) based IA was designed to improve the parameter signal-to-noise ratio (SNR) performance of the system. However, the convergence point of the EXIT chart can not reach the point (1,1) due to the EXIT-optimized IA with the minimal hamming distance \( d_{H, min} = 1 \).

Another category of the ISCD scheme, namely the serial concatenated ISCD scheme, employs the redundant IA to provide the best convergence point (1,1) [5]–[7]. Ashikhmin et al. [9] have shown that inner code rates smaller than 1 cause an inherent capacity loss which cannot be compensated by the outer code (i.e. outer redundant IA). The inner rate-1 codes should be employed in ISCD scheme. This motivate us to adopt the rate-1 recursive non-systematic convolutional (RNSC) encoder. Some redundant IAs proposed by [6]–[8] for the serial concatenated ISCD scheme can obtain the convergence point (1,1) due to \( d_{H, min} > 1 \). The redundant IA [7], [8] using the short block coder is a popular method to generate the \( d_{H, min} \) maximized redundant source codes. In the noisy communication with non-uniform sources, the channel knowledge and the source distribution should be taken into account.

This work considers the serial concatenated ISCD scheme for correlation sources. The scalar quantizer quantizes the sources to generate the quantized symbols, and then the redundant IA generates the redundant symbol with respect to the quantized symbol. The rate-1 RNSC encoder protects the redundant symbols. The end-to-end distortion (EED) of the transmission can provide a design criterion to find the optimal the noisy-channel quantization with IA [10]. Thus, we propose a IA design with the minimized EED value to improve the performance of the serial concatenated ISCD scheme. To efficiently exploit any redundancy remaining in the joint source-channel codes, a modified source decoding algorithm is proposed to improve the extrinsic information exchange of the ISCD. In addition, for self-concatenated convolutional codes, the redundant symbols can be assigned to the quantizer symbols in the de-interleaving domain. Viewing from this perspective, we use the concept of self-iterative decoding algorithm [11], [12] to extract the extrinsic information on the quantizer symbol and the redundant symbol. Through the designed IA, the proposed ISCD with the modified source decoder and the self-iterative channel decoder can further improve the parameter SNR (PSNR) performance of the system.

II. SYSTEM MODEL

Figure 1 shows our transmission model of real-valued, correlation source samples considered in this paper. The input source sequence of \( N \) source samples \( V_i^N = \{v_1, \ldots, v_n, \ldots, v_N\} \) is quantized by the \( Q \)-level scalar quantizer and then each input sample \( v_n \) is mapped to the quantizer symbol \( u_n \). The quantizer’s reproduction level of \( u_n = i \) is denoted by \( c_i \) where \( i \in \mathcal{I} = \{0, 1, \ldots, Q - 1\} \). The index assignment function \( f_{IA} \) generates the redundant symbols by using \( u_n^r = f_{IA}(u_n) \). Therefore, the outputs of the source encoder are the quantizer symbols \( u_n^1 = u_n \) and the redundant symbols \( u_n^2 \). For brevity, the quantizer
Fig. 1. Functional blocks of the transmission system using redundant IA.

Fig. 2. Proposed ISCD scheme with proposed source decoder.

source decoder and \textit{a posteriori} probability (APP) \(P(u_n|Y)\) for each symbol \(u_n\). The \textit{extrinsic} probability \(P_{SD}^{[e]}(u_n)\) will be used to enhance the \textit{a priori} information of the self-iterative decoding [12]. Exchanging \textit{extrinsic} information between the RNSC and the source decoders is iteratively repeated until the incremental reliability gain becomes insignificant. After the last iteration, the APPs are used to determine the MAP signal estimated as follows:

\[
\hat{v}_n = c_{i^*}, i^* = \arg \max_{i \in \{0, \ldots, Q-1\}} P(u_n = i|Y)
\]

III. OPTIMAL REDUNDANT INDEX ASSIGNMENT

Our proposed IA design is based on the EED with considering three crucial factors: 1) the \textit{a priori} knowledge, 2) the artificial correlation provided by the redundant IA, and 3) the hamming distance of the redundancy-aid source codes. Consider a transmission model consisting of a quantizer and an index assignment. The quantizer generates the quantizer symbol \(u_{q,n} = i_n\) and then the redundant symbol is mapped by the IA function as \(u_n = f_{IA}(u_{q,n})\). The symbol-pair is denoted by the notation \(u_n = (i_n, f_{IA}(i_n))\). Prior to entering the channel, each symbol-pair is mapped to the binary-vector. The channel model is characterized by the crossover probability \(p_b\) for redundant IA designing. The \(u_n = (j_n, f_{IA}(j_n))\) represents the output of the channel model. Thus, the symbol-pair transitional probability, namely \(P(u_n'|u_n)\), can be determined by the crossover probability \(p_b\) as follows.

The transitional probability can be determined as

\[
P_{nak}(u_n, u_n') = P(u_n'|u_n) = p_b^{d_H(u_n, u_n')}(1-p_b)^{2*M-d_H(u_n, u_n')},
\]

where the \(d_H(u_n, u_n')\) represents the hamming distance between two bit-vectors of \(u_n\) and \(u_n'\). The precision of the \(P(u_n'|u_n)\) can be enhanced by taking the zero-order \textit{a priori} knowledge (AK0) (i.e. \(P(u_n)\)) and the first-order \textit{a priori} knowledge (AK1) (i.e. \(P(u_{n-1}|u_n)\)) into consideration, and we derive the transitional probability \(P(u_n'|u_n)\) as follows:

\[
P_{ak1}(u_n, u_n') = \frac{P(u_n'|u_n)}{P(u_n)}
= \sum_{u_{n-1}, u_{n-1}'} P(u_{n-1}|u_{n-1}', u_n, u_n') \frac{P(u_{n-1})}{P(u_n)}
= \sum_{u_{n-1}, u_{n-1}'} P(u_{n-1}|u_{n-1}, u_n') P(u_{n-1}'|u_n) P(u_n'),
\]

where the probability \(P(u_n')\) can be computed as \(P(u_n') = \sum_{u_{n-1}} P_{nak}(u_n, u_n') P(u_n)\).

The reconstructed signal is represented by \(\hat{v}_n = c_{j_n}\), where the \(c_{j_n}\) is the quantizer reconstruction level corresponding to the estimated symbol \(j_n\). The EED between the quantized symbol \(i_n\) and the estimated symbol \(j_n\) is denoted by \(D_{i_n,j_n}\).

The total EED can be computed as follows:

\[
EED_{total} = \sum_{i_n,j_n} D_{i_n,j_n},
\]
where \( D_{n,jn} = P_{k1}(u_n, \tilde{u}_n)P(u_n)|c_{n} - c_{jn}|^2 \).

In designing the redundant IA, we consider two constraints: 1) the \( d_{H, min} \) of the codebook of the source code is larger than 1, and 2) the redundant IA is the 1-1 and onto function. The first constraint can improve the performance of ISCD [13]. The IAs [6], [7] may map different quantizer symbols to the identical redundant symbol, and the 2-D source decoder loses some information during extracting the extrinsic information on such quantizer and redundant symbols. To avoid the information loss, we consider the redundant IA function as an 1-1 and onto function.

Specifically, each IA function for \( Q \) level quantizer is defined by \( f_{IA}(i) \), where \( i, f_{IA}(i) \in \{0,1, \ldots, Q - 1\} \), and we have \( f_{IA}(i) \neq f_{IA}(i') \) iff any \( i \neq i' \). Hence the redundant IA design problem can be formulated as the following optimization problem.

\[
\begin{align*}
\text{Minimize} & \quad EED_{total} \\
\text{subject to} & \quad f_{IA}(i) \neq f_{IA}(i') \iff i \neq i', \\
& \forall i, i' \in \{0,1, \ldots, Q - 1\}; \\
& \quad d_{H, min} \geq 2.
\end{align*}
\]

### IV. PROPOSED ISCD

#### A. Two-Dimension Soft-bit Source Decoding Algorithm

In this subsection, we derive a recursive implementation for the proposed Two-Dimension (2-D) source decoder. The proposed source decoder is designed to jointly exploit the natural redundancy of the source encoder, the IA-related redundancy and the channel-related information. The mutual dependency of inter-block adjacent symbol-pairs \( (u_n^1, u_n^2) \) is modeled by a first-order stationary Markov process with 2D-transition probability (i.e. 2-D AK1) \( P(u_n^1, u_n^2|u_{n-1}^1, u_{n-1}^2) \). For the applicability to the ISCD, the algorithm of the proposed 2-D source decoding includes two steps. The first algorithmic step is to compute the 2-D APP \( P(u_n^1, u_n^2|\tilde{Y}) \) for each transmitted symbol-pair \( (u_n^1, u_n^2) \). In the second step, these 2-D APPs are combined with the additional a priori information of the index assignment to extract the extrinsic probabilities \( P^{[c]}_{SD}(u_n^1) \) and \( P^{[c]}_{SD}(u_n^2) \) for both symbols \( u_n^1 \) and \( u_n^2 \), respectively. These extrinsic probabilities are used to improve the a priori information of the channel decoder. Taking the redundant symbols into consideration, the APP for a decoded symbol \( u_n = i \), through receiving code sequence \( \tilde{Y} \), is given by

\[
P(u_n = i|\tilde{Y}) = \begin{cases} 
\sum_{u_n^2 \in f_{IA}(i)} P(u_n^1 = i, u_n^2|\tilde{Y}), & \text{if } u_n \in U_{1,1}^N; \\
\sum_{u_n^1 \in f_{IA}(i)} P(u_n^1, u_n^2 = i|\tilde{Y}), & \text{if } u_n \in U_{2,1}^N,
\end{cases}
\]

where \( IA(i) \) represents the set of symbols assigned from \( i \). In (5), the 2-D APP \( P(u_n^1, u_n^2|\tilde{Y}) \) for each symbol-pair \( (u_n^1, u_n^2) \)

[6] can be determined using Bayes’ theorem as follows:

\[
P(u_n^1, u_n^2|\tilde{Y}) = P(u_n^1, u_n^2|\tilde{Z}, u_n^1, u_n^2)/P(\tilde{Y})
\]

\[
= \alpha_n(u_n^1, u_n^2) \cdot \beta_n(u_n^1, u_n^2) \cdot P(\tilde{Z}|u_n^1, u_n^2)/P(\tilde{Y}),
\]

where \( \alpha_n(u_n^1, u_n^2) = P(u_n^2|u_n^1, \tilde{U}_n^1, \tilde{U}_n^2) \) and \( \beta_n(u_n^1, u_n^2) = P(\tilde{Y}_{n+1}^N|u_{n+1}^1, u_{n+1}^2, \tilde{U}_n^1, \tilde{U}_n^2) \). Using the Markov property of the symbols and the memoryless assumption of the channel, the 2-D forward and 2-D backward recursions of the algorithm can be expressed as

\[
\alpha_n(u_n^1, u_n^2) = P(u_n^1, u_n^2|\tilde{U}_n^1, \tilde{U}_n^2) = \begin{cases} 
\alpha_n(u_n^1, u_n^2) = P(u_n^1, u_n^2|\tilde{U}_n^1, \tilde{U}_n^2), & \text{if } n \leq N; \\
\sum_{u_n^1, u_n^2} \alpha_n(u_n^1, u_n^2), & \text{if } n = N.
\end{cases}
\]

\[
\beta_n(u_n^1, u_n^2) = P(\tilde{Y}_{n+1}^N|u_{n+1}^1, u_{n+1}^2, \tilde{U}_n^1, \tilde{U}_n^2)
\]

\[
= \sum_{u_n^1, u_n^2} \gamma_n(u_n^1, u_n^2) \cdot \beta_n(u_{n+1}^1, u_{n+1}^2),
\]

where the 2-D branch metric \( \gamma_n(u_n^1, u_n^2) \) is expressed as

\[
\gamma_n(u_n^1, u_n^2) = P(\tilde{U}_n^1, \tilde{U}_n^2|u_n^1, u_n^2) = P(\tilde{U}_n^1, \tilde{U}_n^2|u_n^1, u_n^2, \tilde{Y}_{n-1}^N).
\]

Within the iterations the accuracy of the 2-D APP \( P(u_n^1, u_n^2|\tilde{Y}) \) evaluation can be further improved by additional a priori information provided by the SISO channel decoder in terms of its extrinsic probabilities \( P^{[c]}_{CD}(u_n^1) \) and \( P^{[c]}_{CD}(u_n^2) \). Substituting (7) and (9) into (6) leads to

\[
P(u_n^1, u_n^2|\tilde{Y}) = P(u_n^1, u_n^2|\tilde{Y}) \cdot \sum_{u_n^1, u_n^2} \alpha_n(u_n^1, u_n^2) \cdot \beta_n(u_n^1, u_n^2)
\]

\[
+ \sum_{u_n^1, u_n^2} \alpha_n(u_n^1, u_n^2) \cdot \beta_n(u_n^1, u_n^2) \cdot P^{[c]}_{CD}(u_n^1) \cdot P^{[c]}_{CD}(u_n^2).
\]

The decoder’s next step is to compute the symbol-level extrinsic probability \( P^{[c]}_{SD}(u_n^1) \) from the 2-D APPs. By substituting (10) into (5), the symbol APP can be separated into three terms: the channel-related part \( P_{ch}(u_n^1 = i) \), the a priori probability \( P^{[a]}_{SD}(u_n^1 = i) \), and the extrinsic probability \( P^{[c]}_{SD}(u_n^1 = i) \). The symbol APP in (5) for each \( u_n \in U_{1,1}^N \) can be expressed as follows:

\[
P(u_n^1 = i|\tilde{Y}) = \sum_{u_n^2 \in f_{IA}(i)} P(u_n^1 = i, u_n^2|\tilde{Y}) = P_{ch}(u_n^1 = i) \cdot P^{[a]}_{SD}(u_n^1 = i) \cdot P^{[c]}_{SD}(u_n^1 = i),
\]

where the \( P^{[a]}_{SD}(u_n^1 = i) \) is obtained from the de-interleaved sequence of extrinsic probabilities \( P^{[c]}_{SD}(u_n^1 = i) \), and the
\[ P^{[c]}_{SD}(u_1^n = i) \] is expressed as
\[
= \sum_{u_2 \in f_{IA}(i)} P(u_1^n | n = i) \cdot P_{CD}(u_2^n) \\
\cdot \sum_{u_1^{-1}, u_2^{-1}} \alpha_{n-1}(u_1^{-1}, u_2^{-1}) P(u_1^{-1} = i, u_2^n | u_1^{-1}, u_2^{-1}).
\]

Similarly, the \( P^{[c]}_{SD}(u_2^n = i) \) is expressed as
\[
= \sum_{u_1 \in f_{IA}(i)} P(u_1^n | n = i) \cdot P_{CD}(u_2^n) \\
\cdot \sum_{u_1^{-1}, u_2^{-1}} \alpha_{n-1}(u_1^{-1}, u_2^{-1}) P(u_1^{-1} = i, u_2^n | u_1^{-1}, u_2^{-1}).
\]

B. Extrinsic and a priori probabilities in the self-iterative decoding algorithm

By using the BCJR algorithm [14], the channel decoder calculates the APP for each systematic bit \( x_n \) as
\[
P(x_n | Y) = P^{[c]}_{CD}(x_n) \cdot P^{[a]}_{CD}(x_n), \text{ for } x_n \in \{0, 1\},
\]

where \( P^{[c]}_{CD}(x_n) \) is the extrinsic probability of the channel decoder and \( P^{[a]}_{CD}(x_n) \) is a priori probability. The a priori term contains three kinds of information: the extrinsic information of the source decoder \( P^{[c]}_{SD}(x_n) \), the self extrinsic information of SECCC decoder \( P^{[c]}_{CD, self}(x_n) \), and the channel-related information \( P_{ch}(x_n) \). The a priori probability is expressed as
\[
P^{[a]}_{CD}(x_n) = P_{ch}(x_n) \cdot P^{[c]}_{SD}(x_n) \cdot P^{[c]}_{CD, self}(x_n),
\]

where the \( P^{[c]}_{SD}(x_n) \) is obtained from the interleaved sequence of the bit-level extrinsic probabilities \( P^{[c]}_{SD}(u_1^n) \). The bit-level extrinsic probability \( P^{[c]}_{SD}(u_1^n) \) is converted from the symbol-level extrinsic probability as follows:
\[
P^{[c]}_{SD}(u_1^{(m)}) = C \cdot \sum_{u_1 \in \{u_1^{(m)} = 0\}} P^{[c]}_{SD}(u_1^n), \text{ if } u_1^{(m)} = 0;
\]
\[
= \sum_{u_1 \in \{u_1^{(m)} = 1\}} P^{[c]}_{SD}(u_1^n), \text{ if } u_1^{(m)} = 1,
\]

where \( C \) denotes a normalization factor. The self extrinsic information sequence \( \{P^{[c]}_{CD, self}(x_n)\}_{n=1}^{2M-N} \) can be obtained from \( \{P^{[c]}_{CD}(x_n)\}_{n=1}^{2M-N} \).

Within the iterations, the bit-level extrinsic probabilities \( P^{[c]}_{CD}(u_1^{(m)}) \) should be converted to symbol-level extrinsic probabilities. When we assume that all bits within symbol are independent, the symbol-level extrinsic probability for each \( u_n \) can be obtained by the multiplication of bit-level probabilities as
\[
P^{[c]}_{CD}(u_n) = \prod_{m=1}^{M} P^{[c]}_{CD}(u_1^{(m)}).
\]

We have \( P^{[c]}_{CD}(u_1^n) = P^{[c]}_{CD}(u_n) \) and \( P^{[c]}_{CD}(u_2^n) = P^{[c]}_{CD}(u_{N+n}) \). These \( P^{[c]}_{CD}(u_n) \) and \( P^{[c]}_{CD}(u_2^n) \) are used to update the a priori information of the source decoder.

V. SIMULATION RESULTS

Computer simulations were carried out to evaluate the parameter SNR performances given by three ISCD schemes, namely, ISCD1, ISCD2, ISCD3. The ISCD1 scheme consisting of the conventional source decoder [15] and the RSC decoder is designed for the transmission system with the non-redundant index assignment and the rate-1/2 RSC channel code. The ISCD2-3 estimate the source codec parameters with the aid of the redundant index assignment, the residual source redundancy and the rate-1 RNSC channel code. In the ISCD2-3, the proposed source decoder is exploited to compute the APPs \( P(u_1^n, u_2^n | Y) \) and extract the extrinsic information as in Section III. Compared with ISCD2, the ISCD3 employs the self-iterative decoding algorithm to improve the error-correcting capacity of the rate-1 RNSC channel code.

The input signals were first order Gauss-Markov sources with the correlation coefficient \( \rho = 0.95 \). The correlation sources with \( \rho = 0.95 \) provides a good fit to the parameter sequence extracted from the MPEG audio codec [1]. The total of 1000 input source frames are processed by a Lloyd-Max quantizer with scalar level \( Q = 8 \). Three redundant index assignments are used to generate the redundant symbol with respect to the quantizer symbol for the ISCD2-3 schemes. The repetition code based index assignment [6] and the short block code based index assignment [7] are defined by the \( f_{IA1}(\cdot) \) and \( f_{IA2}(\cdot) \), respectively. The \( f_{IA1}(i) = i \), for \( i \in \mathbb{I} \). To construct \( f_{IA2}(\cdot) \), we choose generator matrix \( G = [100101; 010110; 001011] \) and then we obtain \( [u_1^n u_2^n] = [u_1^n(1) \ldots u_1^n(M)] \cdot G, \forall u_1^n \in \mathbb{I} \). Thus, the obtained \( u_2^n \) is represented by \( f_{IA2}(u_1^n) \) for the given \( u_1^n \in \mathbb{I} \) and the sequence \( \{f_{IA2}(i)_i\}_{i=0}^{7} = \{0, 3, 6, 5, 5, 3, 6, 0\} \). The proposed redundant IA is given by \( f_{IA3}(i)_i = \{7, 4, 0, 2, 1, 3, 6, 5\} \). Each symbol is represented by the bit-vector with the length \( M = 3 \). Each of these frames, consisting of \( N = 1000 \) quantizer symbols, is spread by the interleaver and afterwards they were protected by a RNSC encoder with generator polynomial \( G(D) = (1 + D^2)/(1 + D + D^2) \).

The EXIT chart is a useful tool to analyze the converge behavior of the ISCD. Figure 3 plots the EXIT charts of the serial ISCD with the redundant IA and the serial ISCD with non redundant IA. The \( I_{ASD} \) and \( I_{ACD} \) represent the average a priori information for source decoding and channel decoding, respectively. The average extrinsic information for source decoding and channel decoding is denoted by the \( I_{ESD} \) and \( I_{ECD} \), respectively. We can observe that the maximal \( I_{ESD} \) of the conventional source decoder [15] can be improved through EXIT-optimized IA [4]. However, the theoretical converge points of the EXIT charts are bad at the channel SNR = -6 dB. By exploiting the 2D source decoder with the redundant IA, the converge point can be improve to the point (1,1) at the channel SNR = -6 dB. This indicates that the proposed 2D source decoder allows the serial ISCD.
to achieve the high performance through the IA2 and IA3 at the channel SNR = -6 dB. Figure 4 plots the parameter SNRs associated with ISCD1-3 schemes. The parameter SNR is calculated according to $\sum_n v_n^2 / \sum_i (v_n - \bar{v}_n)^2$. The ISCD2 with IA1 obtains the lowest parameter SNR because the IA1 can not provide more useful a priori knowledge for source decoding. By employing the IA2 to convey the extra a priori knowledge for 2D source decoder, the ISCD2 achieves the higher parameter SNR than the ISCD1. The ISCD3 uses the self-iterative decoding algorithm as described in Section IV-B to enhance the extrinsic information exchange. Thus, the ISCD3 outperforms ISCD1-2 schemes. Furthermore, the ISCD3 with the proposed IA further improves the parameter SNR performance in comparison with the IA2.

VI. CONCLUSION

This study presents an iterative source-channel decoding algorithm which effectively increases the performance of the joint source-channel coder with the redundant IA and rate-1 channel coder. We proposed a 2-D source decoder which can jointly exploit the artificial redundancy from the index assignment and the inherent redundancy remaining in the source encoder. The proposed ISCD scheme with 2-D source decoder employs the self-iterative decoding algorithm to improve the extrinsic information exchange. An EED-based redundant IA design was proposed to increase the useful additional redundancy for the ISCD. Simulations show that the proposed ISCD outperforms the conventional iterative decoding schemes. Furthermore, the parameter SNR performance of the proposed ISCD can be further improved through the optimized redundant IA.

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