A Low Complexity Antenna Selection Algorithm for Energy Efficiency in Massive MIMO Systems

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Abstract—Massive multi-input multi-output (MIMO) systems have a great potential to improve the capacity without increasing system bandwidth or transmission power for the wireless communications [1]. To tackle the tremendous demand for cell throughput and provide high-quality link for cell edge users, massive MIMO is a promising technique. The work in [2] has shown that when the number of antennas at base station (BS), \(N\), is large, the transmission power of each user can be proportionally reduced to \(1/N\) with perfect channel state information (CSI) at BS. Although the transmission power is reduced in massive MIMO systems, the circuit power consumption (power consumption in radio frequency (RF) chains) would be gradually increased by the number of antennas due to the increased size of hardware. The work in [3] demonstrates that the circuit power consumption should be taken into account when the energy efficiency is accessed in massive MIMO systems. In [4], the authors consider the RF chain configuration to maximize the spectral efficiency under the total power constraint in massive MIMO systems. Therefore, when the number of antennas grows extremely large, antenna selection can be adopted as an approach to decide the RF chain configuration to enhance the spectral efficiency or energy efficiency in massive MIMO systems.

The point-to-point MIMO systems with antenna selection have been extensively studied in the literatures [5]-[14]. In [5], Molisch et al. derives an upper bound of the channel capacity with antenna selection at the receiver side. In [6], it is shown that antenna selection applied to the ill-conditioned channels can increase the channel capacity. In [7], Heath et al. proposes an antenna selection algorithm to minimize the error rate in spatial multiplexing systems with linear receivers. Antenna selection techniques in conjunction with the space-time coding are presented in [8] and [9]. Also, antenna selection method can be applied to spatial modulation (SM) and space-shift keying (SSK) systems in [10] and [11], respectively. In [5] and [12]-[14], the authors present antenna selection algorithms with selection criterion for choosing the antenna subsets that maximize the channel capacity.

The exhaustive search considering all possible antenna subsets gives the optimal solution. However, it becomes computationally prohibitive when the number of available antennas increases. For this reason, suboptimal antenna selection algorithms with lower complexity are needed. A simple selection algorithm called the norm-based selection (NBS) method chooses the receive antennas corresponding to the rows of the channel matrix with the largest Euclidean norms [5]. The NBS method has extremely low complexity but it does not perform well at the high signal-to-noise ratio (SNR) regime or under ill-conditioned channels. An alternative selection algorithm namely the correlation-based selection (CBS) method has been proposed in [12]. This method considers the correlation between any two rows of the channel matrix. It searches the two rows with the highest correlation and removes the one with the smaller norm. In [13] and [14], the authors proposed the antenna selection algorithms with selection rules to maximize the channel capacity.

In conventional antenna selection methods, the authors have not considered the circuit power consumption in their studies. Hence, they try to select the antenna subset to maximize the channel capacity under given transmission power. However, the works in [15] and [16] have shown that the antenna selection under the effect of circuit power consumption is very different from the conventional works. Jiang and Cimini in [15] explore to maximize the energy efficiency with antenna selection in single stream MIMO systems where the power consumed by RF chains is considered. The case of multi-stream is also investigated by [16].
In this paper, we consider the problem of transmit RF chain configuration aiming to maximize energy efficiency under ill-conditioned channels in massive MIMO systems. The RF chain configuration is decided by applying antenna selection methods. Considering the trade-off between performance and complexity, we propose the norm-and-correlation-based selection algorithm for energy efficiency maximization. Our selection metric considers the effect of the norm of each column and correlation between columns where channel Gram matrix can be used to reduce the computational complexity.

The remainder of our paper is organized as follows. In Section II, the system model is described. In Section III, we formulate the problem of transmit RF chain configuration for energy efficiency and propose the antenna selection algorithm based on norm and correlation. In Section IV, we analyze the complexities of our proposed algorithm and other algorithms. Finally, simulation results and conclusions are presented in Section V and Section VI, respectively.

II. SYSTEM MODEL

We consider a point-to-point MIMO system. The transmitter and the receiver are equipped with $N_T$ transmit antennas and $N_R$ receive antennas, respectively. We assume that the transmitter and the receiver have the same number of RF chains as shown in Fig. 1. Each RF chain is connected with one transmit or receive antenna. Suppose the channel is flat and slow fading. The received signal vector is then given by

$$y = Hx + n$$

where $H \in \mathbb{C}^{N_R \times N_T}$ is the channel matrix and $x \in \mathbb{C}^{N_T \times 1}$ is the transmitted signal vector. The $n \in \mathbb{C}^{N_R \times 1}$ is the additive white Gaussian noise vector whose entries are i.i.d. circular symmetric complex random variables with zero mean and variance $\sigma_n^2$. In this paper, to evaluate the performance for correlated channels, we assume the correlation exists in the transmitter and the receiver. According to the Kronecker model, the channel matrix is represented as

$$H = R_T^{1/2} H_\omega R_T^{1/2}$$

where $H_\omega \in \mathbb{C}^{N_R \times N_T}$ is the matrix whose entries are i.i.d. circular symmetric complex Gaussian random variables with zero mean and unit variance. $R_T \in \mathbb{C}^{N_T \times N_T}$ and $R_R \in \mathbb{C}^{N_R \times N_R}$ are the correlation matrices among $N_T$ transmit antennas and $N_R$ receive antennas, respectively, where the correlation is assumed to exponentially decrease by antennas spacing. Then, the transmit correlation matrix is defined by

$$R_T = \begin{bmatrix}
1 & \alpha_t & \cdots & \alpha_t^{N_T-1} \\
\alpha_t & 1 & \cdots & \alpha_t^{N_T-2} \\
\vdots & \vdots & \ddots & \vdots \\
\alpha_t^{N_T-1} & \alpha_t^{N_T-2} & \cdots & 1
\end{bmatrix}$$

where $\alpha_t$ is the correlation coefficient of transmit antenna. The representation of $R_R$ is the same as (3) with correlation coefficient $\alpha_r$. We assume that the CSI is unknown at the transmitter, and the power is equally allocated among the transmit antennas. The capacity $C(H)$ of instantaneous CSI is given by

$$C(H) = \log_2 \det \left( I_{N_R} + \frac{\rho}{N_T} HH^H \right) = \log_2 \det \left( I_{N_T} + \frac{\rho}{N_T} H^H H \right)$$

where $\rho = \frac{P_t \cdot \beta}{\sigma_n^2}$ is the average SNR at each receive antenna, $P_t$ is the transmission power, $\beta$ is the propagation loss, $I_N$ is the $N \times N$ identity matrix, $\det(\cdot)$ is the matrix determinant, and $(\cdot)^H$ is the Hermitian transpose.

The total power consumption at the transmitter and the receiver is denoted as

$$P_{total} = \frac{P_t}{\eta} + k_t P_{ct} + k_r P_{cr} + 2P_{syn}$$

where $\eta$ is the efficiency of power amplifier (PA), $k_t$ and $k_r$ stand for the number of active transmit and receive RF chains, respectively; $P_{ct}$ and $P_{cr}$ represent the circuit power consumptions at each transmit RF chain and receive RF chain, respectively; and $P_{syn}$ is the power consumed by the local oscillator. In general, there is a single oscillator used for frequency synthesis at the transmitter and receiver sides, respectively. The first term of (5) represents the power consumption at the PA, which depends on the transmission power radiated from the transmit antenna. Both circuit power consumptions of transmit and receive RF chains can be regarded as a fixed value, which is independent from transmission power [17].

III. PROBLEM FORMULATION AND ANTENNA SELECTION ALGORITHM

Assume the total power at the transmitter and the receiver is limited, i.e., $P_{total} \leq P_{max}$. The maximum transmission power when activating $k_t$ transmit RF chains and $k_r$ receive RF chains is given by

$$P_{t,\max} = \eta(P_{max} - k_t P_{ct} - k_r P_{cr} - 2P_{syn})$$

Here, we only consider the transmit RF chain configuration and all receive RF chains activated. We denote $\omega$ as the subset of transmit antennas and the cardinality of $\omega$, i.e., $|\omega|$ represents the number of active transmit RF chains. According to the definition of energy efficiency which is spectral efficiency
based selection metric given by

$$\text{EE} = \frac{C(H_L)}{\eta + |\omega|P_{ct} + N_R P_{cr} + 2P_{syn}}$$

(7)

where $C(H_L)$ is the capacity of selected transmit antennas corresponding to the number of active transmit RF chains. The $H_L$ is the sub-channel matrix consisting of selected columns of $H$. To maximize energy efficiency in (7) under $P_{max}$, it can be formulated by the optimization problem given by

$$\max_{\omega \in \Omega} \text{EE}$$

subject to $\Omega$ is the set of all transmit RF chain configurations.

Our goal is to maximize the energy efficiency. Under the total power constraint, maximizing the energy efficiency is equivalent to maximizing the capacity. Hence, the optimization problem can be rewritten as

$$\max_{\omega \in \Omega} C(H_L)$$

subject to $\frac{P_t}{\eta} + |\omega|P_{ct} + N_R P_{cr} + 2P_{syn} \leq P_{max}$.

(8)

Due to the total power constraint, we should consider the trade-off between the transmission power and the circuit power consumption of transmit RF chains. The more transmit RF chains the transmitter activates, the less transmission power the transmitter can radiate. The optimal solution can be obtained by searching all possible configurations, i.e., exhaustive search. However, the cardinality of $\Omega$ is $2^{N_T} - 1$ which is very large when the number of available RF chains grows. In order to reduce the huge search complexity, the greedy search algorithm can be considered as a viable alternative.

In this section, we consider a MIMO system adopting transmit antenna selection. Assume the CSI is perfectly known at the receiver, where the selection is implemented and the selected transmit antenna subset is fed back to the transmitter. The procedure is as follows: 1) consider the configuration subset of fixed number of active RF chains; 2) search the antenna subset maximizing specific selection rule over set of the given number of active RF chains. According to [12], we know that $H_n$ has most uncorrelated columns with maximum norms. Considering the trade-off between performance and complexity, we propose a low complexity metric for antenna selection with little performance degradation compared with [13]. The metric combines both effects of norms of columns and uncorrelation between columns. Note that $\theta_{i,j}$ and $\phi_{i,j}$ are the correlation and uncorrelation coefficients between the $i$-th column and $j$-th column of channel matrix $H$, respectively. Let us define the following variables:

$$\theta_{i,j} = \frac{|h_i^H h_j|}{\|h_i\| \|h_j\|},$$

(11)

$$\phi_{i,j} = \sqrt{1 - \theta_{i,j}^2} = \sqrt{1 - \frac{|h_i^H h_j|^2}{\|h_i\|^2 \|h_j\|^2}},$$

(12)

Therefore, we propose the selection metric defined as

$$g_{n,i} = \begin{cases} \|h_i\|, & \text{if } n = 1 \\ \frac{\|h_i\|^2}{\gamma} + \sum_{m=1}^{n-1} \phi_{w_{m,i}}, & \text{if } n > 1 \end{cases}$$

(13)

where $g_{n,i}$ is the cost function of $i$-th column in the $n$-th step, $\gamma$ is the normalized factor ($\gamma = \sqrt{N_R}$), and $w_{m,i}$ is the index of $m$-th step selected antenna. Our metric considers the norm of each column and correlation between columns, and can be obtained via computing the channel Gram matrix $G$ given by

$$G = H^H H = \begin{bmatrix} \|h_1\|^2 & h_1^H h_2 & \cdots & h_1^H h_{N_T} \\ h_2^H h_1 & \|h_2\|^2 & \cdots & h_2^H h_{N_T} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_T}^H h_1 & h_{N_T}^H h_2 & \cdots & \|h_{N_T}\|^2 \end{bmatrix}.$$  

(14)

By applying (14), whose entries contain the information of norm and correlation, the computational complexity can be reduced in obtaining the selection metric.

The proposed selection algorithm follows the greedy principle. The proposed selection algorithm adopts an incremental procedure meaning that it starts with the empty set of available antennas and then adds one antenna to the set per step. In each step, we choose the column which makes the proposed metric largest. Therefore, the goal is to find the transmit antenna index $w_n$ which attains the maximal cost function in each step. The selection rule is as follows:

$$w_n = \arg \max_{i \in \mathcal{I}} g_{n,i}$$

(15)

where $\mathcal{I}$ is the set of not-selected transmit antennas. In each step, the set of selected transmit antennas subset is generated and then the energy efficiency is calculated based on this set. Finally, we compare the energy efficiency of each step and choose the antenna subset that attains the largest energy efficiency as the solution. The proposed algorithm is summarized in Table I.
| TABLE I  
| PROPOSED SELECTION ALGORITHM. |

| Initialization: compute (14) and set $I = \{1, 2, ..., N_T\}$ and $W = \emptyset$. |
| Matrix $A(a, b)$: element of $a$-th row and $b$-th column of $A$. |
| for $n = 1 : N_T$ |
| if $n = 1$ |
| for all $i \in I$ |
| $g_{n, i} = \sqrt{G\left(t, i\right)}$ |
| end |
| $w_n = \arg \max_{i \in I} g_{n, i}; I = I - \{w_n\}; W = W + \{w_n\}$ |
| else |
| for all $i \in I$ |
| $\phi_i = 0$ |
| for $m = 1 : |W|$ |
| $\phi_m = \phi_i + \sqrt{1 - \frac{G\left(t, W\left(m\right)\right)\cdot G\left(W\left(m\right), W\left(m\right)\right)}{G\left(t, i\right)\cdot G\left=W\left(m\right), W\left(m\right)\right)}$ |
| end |
| $g_{n, i} = \sqrt{\frac{G\left(t, i\right)\cdot \sqrt{W\left(m\right)\cdot W\left(m\right)\right}}{N_R}} + \phi_i$ |
| end |
| $w_n = \arg \max_{i \in I} g_{n, i}; I = I - \{w_n\}; W = W + \{w_n\}$ |
| end |
| Calculate $E_{E_n}$ |
| $E_{E_n}^T = \max_{(1, 2, ..., N_T)} E_{E_n}$ |

| IV. COMPLEXITY ANALYSIS |

In this section, we compare the complexity with different algorithms by counting the numbers of floating-point operation (FLOP) [18]. A FLOP means a simple complex floating point operation such as an addition, subtraction, multiplication, or division. The antenna selection algorithms can be categorized into two groups: incremental selection algorithm (ISA) and decremental selection algorithm (DSA). In ISA, it starts with the empty set of antennas and add one antenna into the set in each step. In contrast, in DSA, it begins with the full set of available antennas and one antenna is removed from the set per step. The complexity of different antenna selection algorithms are compared as follows: capacity-based selection, CBS, NBS, and proposed selection algorithm. The goal of capacity-based selection algorithm is to select one more antenna, which results in the highest increase of capacity, in each step. The objectives of CBS and our proposed algorithms are to make the columns of sub-channel matrix minimally correlated. The NBS algorithm selects the antennas with the largest antenna gains. Our proposed selection algorithm and capacity-based selection algorithm belong to ISA, and CBS algorithm is in the DSA. For ISA, the selection metric can be divided into two parts: initial antenna selection ($n = 1$) and $n = 2 \sim (N_T - 1)$ where $n$ represents the selection step. The total flop count of antenna selection algorithm can be obtained by the sum of flop counts of each part. Specifically, the NBS algorithm only needs to compute the norm of each column of channel matrix $H$. The total flop count for NBS algorithm is $2N_TN_R - N_T$. In CBS algorithm, according to [12], the correlation of CBS is defined as $|\langle x, y \rangle|$ where $\langle x, y \rangle$ represent the inner product between vector $x$ and $y$. When performing the CBS algorithm, it searches two columns with the highest correlation within $H$ and then removes one of them with smaller norm in each step. However, CBS algorithm can use channel Gram matrix to acquire information about norm and correlation. Generating channel Gram matrix requires $N_R N_T^2 + N_R N_T - N_T^2 - \frac{N_R^2}{2} - \frac{N_T}{2}$ flops based on [18]. The flop count of $|\langle h_i, h_j \rangle|$ is $\sqrt{h_i^H h_j} |h_i^H h_j|^H$ is 2 because the calculation of $|\langle h_i, h_j \rangle|$ only contains one multiplication and square root. Since $(N_T - n - 1)$ transmit antennas remain in $n$-th step, the flop count required for selecting antennas to remove is 2 $(N_T - n + 1)$ where $\binom{N}{K}$ represents $K$ elements chosen from a set of $N$ elements. The total flop count of CBS algorithm is

$$N_R N_T^2 + N_R N_T - \frac{N_T^2}{2} - \frac{N_T}{2} + \sum_{n=1}^{N_T-1} \binom{N_T - n + 1}{2}.$$  \hspace{1cm} (16)

In capacity-based selection algorithm, the initial complexity for calculating the Frobenius norm over $N_T$ transmit antennas requires $2N_TN_R - N_T$ FLOPs. Before computing the complexity in each step, we let $R = \frac{n \sigma^2}{\sum_{k=0}^{m-1} I_{N_k} + H_{n-m}H_{n-m}^H}$ first. In each step, computing $R$ requires $(2n-3) \left(\frac{N^2 + N_n}{2}\right) + N_R + 8$ FLOPs and the inverse of $R$ requires $N^3 + N_R + N_T + (N_T - n + 1) \left(\frac{3}{2} N_R^2 + \frac{3}{2} N_R - 1\right)$. Hence, the flop count of capacity-based selection algorithm is

$$2N_TN_R - N_T + \sum_{n=2}^{N_T-1} \left(\binom{N_T - n + 1}{2} \frac{N_R^2 + N_R}{4} + N_R + \frac{N_T}{2} + \frac{N_T}{2} + (N_T - n + 1) \left(\frac{3}{2} N_R^2 + \frac{3}{2} N_R - 1\right)\right).$$ \hspace{1cm} (17)

In proposed selection algorithm, the (14) is computed first. The flop count required for (14) is $N_R N_T^2 + N_R N_T - \frac{N_T^2}{2} - \frac{N_T}{2}$. The Frobenius norms of columns over $N_T$ transmit antennas can be obtained from the diagonal entries of (14). For our metric, the flop count of $\frac{|h_i|}{\sqrt{N_R}}$ and $\phi_{i,j} = \sqrt{1 - \frac{h_i^H h_j h_j^H h_i}{|h_i|^2 |h_j|^2}}$ is 2 and 5, respectively. The calculation of $h_i^H h_j$, $h_i^H h_i$, $|h_i|^2$, and $|h_j|^2$ can be obtained from (14). Therefore, the computation for them is not required. The flop count of $\frac{|h_i|}{\sqrt{N_R}} + \sum_{m=1}^{N_T-1} \phi_{w,m}$ required in $n$-th step is $6n - 5$. Since there are $(N_T - n - 1)$ transmit antennas in $I$ in $n$-th step, the flop count required for selecting $h_i$ is $(N_T - n + 1) (6n - 5)$. Hence, the flop count of our proposed algorithm is

$$N_R N_T^2 + N_R N_T - \frac{N_T^2}{2} - \frac{N_T}{2} + \sum_{n=2}^{N_T-1} (N_T - n + 1) (6n - 5).$$ \hspace{1cm} (18)
The comparison of flop counts for different number of transmit antennas is given in Fig. 2. From Fig. 2, we can see that the complexity of the proposed selection algorithm is much lower than the capacity-based selection algorithm, since the capacity-based selection metric contains the operation of matrix inversion. Although the complexity of the proposed selection algorithm is higher than CBS and NBS algorithms, the performance of the proposed selection algorithm is better.

V. SIMULATION RESULTS

In this section, we provide the simulation results under ill-conditioned channels to validate the proposed selection algorithm. According to [4], the propagation loss is modeled as $\beta = G_0 (d_0/d)^\kappa$, where $d_0$ is the reference distance; $\kappa$ is the path-loss exponent; $G_0$ is the path loss at the reference distance $d_0$ and given by $G_0 = c^2/(4\pi d_0 f_c)^2$, where $c$ is the speed of light and $f_c$ is the carrier frequency. The noise power at receiver is determined as $\sigma_n^2 = N_0 B N_f$, where $N_0$ is the power spectral density of additive white Gaussian noise; $B$ is the system bandwidth; $N_f$ is the noise figure. The parameters adopted from [19] are summarized in Table II. For comparison, we show the performance with different antenna selection algorithms: capacity-based selection in [13], CBS in [12], NBS in [5], proposed selection algorithm, and no selection which means all transmit and receive antennas are activated.

We show the energy efficiency as a function of distance $d$. In Figs. 3 and Fig. 4, we compare the performance of proposed selection algorithm and other antenna selection algorithms under different transmission distance. Due to the total power constraint, we should consider the trade-off between the transmission power and circuit power consumption of transmit RF chains. The more transmit RF chains the transmitter activates, the less transmission power the transmitter radiates. We can see that the case of all transmit and receive antennas activated gives the poorest performance. Hence, turning off part of the transmit RF chains is needed. The performance of the capacity-based selection algorithm achieves the better performance, since its selection metric is based on capacity maximization, but its complexity is high. In Fig. 3, the proposed selection algorithm outperforms the NBS and CBS algorithms. The reason for NBS algorithm’s poorer performance than other antenna selection algorithms is that it does not consider the effect of correlation. For two antennas with high gains but with high correlation, the multiplexing gain vanishes due to high correlation. The performance of CBS algorithm is also poor because CBS algorithm does not consider the effects of norm and correlation simultaneously. Note that the performance of the proposed selection algorithm is comparable with the capacity-based selection algorithm, while the complexity of the proposed selection algorithm is much lower than its complexity. Fig. 4 represents the case of moderate transmit and receive correlation coefficients. The performance gap between NBS algorithm and other selection algorithms is shortened. We cannot see the advantage of the
proposed selection algorithm over the NBS algorithm. The reason is that the NBS algorithm is appropriate when the channel is i.i.d.

Fig. 5 and Fig. 6 illustrate the case of fixed transmission power. The total power consumption increases with the number of selected transmit antennas. We show the energy efficiency versus the transmission distance \(d\). We compare the performance of proposed selection algorithm and other selection algorithms. In these cases, the performance of the proposed algorithm is also very close to that of capacity-based selection algorithm and better than NBS and CBS algorithms in all transmission distances. Because the correlation coefficients in Fig. 5 are higher than the case in Fig. 6, the performance gap is more obvious. We can observe when all transmit and receive antennas are activated, the energy efficiency is worse than adopting antenna selection. Performing the antenna selection can improve the energy efficiency in massive MIMO systems.

VI. CONCLUSIONS

We propose the antenna selection algorithm with low complexity for energy efficiency in massive MIMO systems. Our selection metric is based on the norm and correlation where channel Gram matrix can be used to reduce the computational complexity. As compared with capacity-based selection method, the proposed selection method provides lower complexity. By effects of circuit power consumption, the activation of all transmit and receive antennas attains poorer energy efficiency in massive MIMO systems. Applying antenna selection approach for the RF chain configuration can significantly increase the energy efficiency. Through simulation results, we demonstrate that the proposed algorithm outperforms NBS and CBS methods under ill-conditioned channels, and achieves comparable performance and lower complexity when compared with capacity-based selection algorithm.

REFERENCES